Directed Coverage in Wireless Sensor Networks: Concept and Quality

Xiaole Bai*, Lei Ding*, Jin Teng†, Sriram Chellappan‡, Changqing Xu† and Dong Xuan*

* *Dept. of CSE*
The Ohio State Univ.
Columbus, Ohio, USA
{baixia, dinglei, xuan}@cse.ohio-state.edu

† *Dept. of EE*
Shanghai Jiao Tong Univ.
Shanghai, P. R. China
{kenteng, cqxu}@sjtu.edu.cn

‡ *Dept. of CS*
Missouri Univ. of Science and Technology
Rolla, MO, USA
chellaps@mst.edu

Abstract

In this paper, we introduce a new type of coverage for wireless sensor networks, called Directed Coverage *(D-Coverage). Basically, D-Coverage is the coverage provided by a sensor network monitoring an area between two boundaries, through which the intruder attempts to penetrates the area. We also study how to measure the quality of D-Coverage. Our first evaluation approach is a projection-based simple approach, while our second approach is a more comprehensive Markov chain based approach. Our evaluation approaches can accurately evaluate the quality and provide good guidelines for sensor network deployment and run-time repair.*

1. Introduction

In the recent past, there have been a number of documented instances of physical intrusion threats to many sensitive facilities. The critical feature of such threats is that the goal of the intruder is to not just wander around in the facility vicinity, but to rather *penetrate through* the boundaries of the facility. Under such threats, it is critical to detect the penetrators (i.e., before they successfully penetrate the facility) in a reliable manner.

In this paper, we introduce a new concept called *Directed Coverage* (or *D-coverage* in short) for wireless sensor networks. In D-coverage, the area covered by sensor nodes has two boundaries, i.e. source boundary and destination boundary. The intruder penetrates the area starting from the source boundary and exits at the destination boundary. The area of D-coverage as well as its corresponding source and destination boundaries can be in any shape. The concept of D-coverage is general. It encompasses a wide spectrum of applications. Barrier coverage studied in [2][4][15][9] is a special cases of D-coverage. D-coverage exhibits a strong directional tendency due to two factors: 1) the existence of source and destination boundaries and 2) the intention of the intruder to penetrate the network. This directional tendency of the penetrator has significant impacts to both the required sensor network deployment and run-time network repair and configuration. In providing D-coverage, deployers can incur significant savings in number of sensors needed if such tendencies are exploited at sensor network deployment time. Also, a significant amount of energy savings can be realized by considering direction-oriented nature of the penetrator movements during sensor node wake-up and sleep scheduling.

Clearly, the quality of coverage, which reflects how good a sensor network detects the intruder is a critical issue. As stated above, barrier coverage is a special case of D-coverage. In [2][7][8][15], absolute barrier coverage is studied where all the intruders are to be detected with probability 1. However, absolute D-coverage (including absolute barrier coverage) is not always feasible because: 1) wireless sensors are of dynamic nature, and may fail unexpectedly. Also, due to limited energy, they are often scheduled into duty circles of being awake and asleep in turn; 2) sensor deployment is often constrained by geographical or technical limitations. Some areas may be necessary for detection but not feasible for deployment; 3) in many cases, redeployment is not feasible for sensor nodes replacement; 4) it may be also too expensive to achieve absolute D-coverage when the deployment area is large. Hence, D-coverage with less than 100% guaranteed penetration detection, referred in this paper as *probabilistic D-coverage*, is more practical and deserves a systematic study.

Due to the special directional features and irregular boundaries in D-coverage, existing evaluation metrics, e.g., in [3][4][16], cannot be applied. In this paper, we propose two evaluation approaches for D-coverage: a projection-based simple approach, and a Markov chain based comprehensive approach. The projection-based approach is simple, fully utilizing the directional feature of D-coverage, when the movement of penetrators can be considered as a straight line. The Markov chain approach is comprehensive and can more accurately evaluate the quality at the cost of complexity.

The rest of the paper is organized as follows. Instead of a separate section for related works, we refer to several other works related to ours at different places

978-1-4244-5113-5/09/$25.00

in the paper. In Section 2, we present the concept of D-coverage and its basic definition. In Section 3, we present our approaches for evaluating the quality of D-coverage. We present our performance evaluations in Section 4, and finally conclude our paper in Section 5.

2. The Concept of Directed Coverage

In this section we introduce the concept of directed coverage, its features and its relationship with other types of coverage defined in previous literatures.

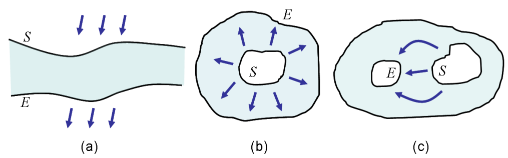

Figure 1. (a) Surveillance to protect national borders. (b) Surveillance to secure a detention facility. (c) Surveillance to detect army movements between units.

In Fig. 1(a), we show a scenario of protecting national borders, where there are two distinct source and destination boundaries. In between these two boundaries, the intruder can potentially take any direction to move (although its objective is to also rapidly penetrate the boundaries, than just wander there). In Fig. 1(b), we show a scenario of a detention facility (inside the smaller circle), from where escape attempts by prisoners need to be detected. In this case, the inner boundary is the source, while the outer boundary is the destination which prisoners attempt to reach. In Fig. 1(c), we show an army scenario where each non-shaded area is an independent army unit. The goal is to detect soldier movements between units. In this case, each boundary is itself a source and a destination.

We consider the problem of a sensor network that can detect the penetration between boundaries of a network. For the rest of the paper, we call the boundary from which the penetrator enters the network the *Source Boundary*, denoted by S. We call the boundary from which the penetrator exits the network as *Destination Boundary*, denoted by E. The boundaries could be anything from a simple geometric shape (like a straight line) or irregular.

Definition 2.1: Directed Coverage (D-coverage) The coverage provided by a sensor network in detecting penetrators starting from any point at S before it penetrates through any point at E.

Although the penetrator may take any direction at one instant, overall, it moves towards the destination boundary. The presence of the boundaries and the penetrator's intention typically constrain the penetrators movement paths and show a strong directional tendency. Fig. 2 shows a typical instance.

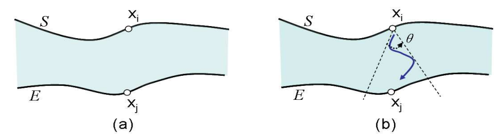

Figure 2. (a) Assume the penetrator enters at point x_i at the source boundary (S) and penetrates the network at point x_j at the destination boundary (E). It is unlikely that the penetrator will move arbitrarily between these two points. (b) Its movement area is constrained.

A pertinent concern to raise at this point is the relationship between D-coverage with other coverage concepts proposed in the literature. As we discussed in Section 1, barrier coverage as studied in [2][4][15][9] is a special case of D-coverage, where the coverage area is a barrier with two parallel boundaries. There are also a vast of works that typically consider monitoring events occurring at arbitrary points in the networks and/ or intruders that are expected to arbitrarily move in the networks (recognizing and recording trace of an objective), e.g. trap coverage in [5]. The difference lies on the fact that the concept of D-coverage focuses on the feature of directional tendency while they do not.

3. The Quality of Directed Coverage

The traditional quality metrics used for monitoring and tracking sensor network cannot be directly applied to evaluate D-coverage.

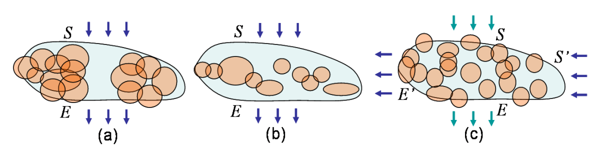

Figure 3. Sensors are deployed in the same area in different ways as shown in (a), (b) and (c). Intruders penetrate the network along straight lines with directions shown in arrows.

For instance, the ratio of the covered area size over the total area size is widely adopted [16][2][12] etc. to measure the quality of full area coverage. However, this metric cannot present an accurate view for the quality of penetrator surveillance. Fig. 3 (a) and (b) show a network of active sensors (their sensing ranges), wherein the penetrator crosses the network from its top boundary to its bottom boundary. Clearly, Fig. 3 (a) has a relatively poorer quality of penetrator surveillance, while possessing a higher quality of the traditional metric of coverage ratio. A metric for tracking quality is ALUL (Average Linear Uncovered Length) [3], which is the average length of uncovered paths traveled by an penetrator starting from a random (uniformly chosen) location within the field. ALUL does not explicitly consider the penetrator's direction as well as the relative position of source and destination boundaries. Thus it is not able to accurately reflect the quality of D-coverage. As shown in Fig. 3(c), different penetration directions will clearly have different detection probability while ALUL possesses only one value. The existing metric for barrier coverage provided in [4] is hard to be generalized for irregular boundaries and arbitrary penetration directions.

Our evaluation for D-coverage is designed to meet the following requirements: 1) it should be able to consider both absolute and probabilistic D-coverage; 2) it should be general enough to handle complicated scenarios, e.g., irregular sensing ranges, irregular sensor deployment areas and non parallel penetrating paths; 3) it should be backed by well studied background to facilitate extension, optimization, and algorithm design.

3.1. Preliminaries

We assume that penetrators are not aware of sensor locations, and hence will neither be able to tune their movement strategy based on sensor location nor destroy the sensors.

Notice that, though the sampling rate may be fast, local detection decision can be relatively slow. For instance, the sampling interval for acoustic sensors is commonly 500us. For PIR or Magnetometer, it is usually 10ms. However, local detection usually is based multiple samples and needs $0.1 \sim 5$s for them [1]. In surveillance applications, penetrators possesses a certain speed. When coupled with the sampling intervals of sensors, an intersection within the sensing ranges cannot practically constitute penetrator detection. It is more reasonable to expect that a detection occurs only when the intersection between a penetration trace and the sensing ranges achieves a certain length. We denote such length by D.

We define $\mathcal{L}$ as a set of paths $\mathcal{L} = \{l_1, l_2, l_3, \cdots, l_n\}$ that start from the source boundary S and end at the destination boundary E. For any path $l_i \in \mathcal{L}$, we denote $P[l_i]$ as the probability of the penetrator takes path l_i. Its value is decided by the penetrator movement pattern. We denote the number of times the penetrator is detected by different sensors along path l_i by $\mathbb{D}^s(l_i)$, and the maximum number of detections made simultaneously by different sensors by $\mathbb{D}^c(l_i)$.

Definition 3.1: Concurrent λ-K D-Coverage It is achieved when $\sum_{\ell_i \in \mathcal{L}} P[l_i] P[\mathbb{D}^c(\ell_i) \geq K] \geq \lambda$, $\lambda \in [0, 1]$.

Definition 3.2: Sequential λ-K D-Coverage It is achieved when $\sum_{\ell_i \in \mathcal{L}} P[l_i] P[\mathbb{D}^s(\ell_i) \geq K] \geq \lambda$, $\lambda \in [0, 1]$.

Concurrent λ-K D-coverage states detection occurs only when the penetrator has been detected *simultaneously* by K different sensors. In sequential λ-K D-coverage, simultaneousness being unimportant, detection occurs as long as the penetrator has been already detected by K different sensors. We comment that the concept of directed coverage is always coupled with *paths* here.

Parameter λ can be viewed as a performance metric to reflect the quality of D-coverage. While there could be other possible metrics, we believe that based on our definition, λ is a natural and direct metric capturing the essentials of penetration detection in sensor networks. A large λ implies the penetrator is more likely to be detected along its path, thereby indicating a better quality of the directed coverage. Note that in absolute directed coverage $\lambda \equiv 1$, while in probabilistic one $\lambda \in [0, 1]$. The works in [15] and [2] both focus on the case of $\lambda \equiv 1$, and not otherwise.

3.2. Projection Based Simple Approach

This approach is based on the assumption that the movement pattern of the penetrator can be considered as straight lines.

For any straight path $\ell_i \in \mathcal{L}$, we can always find a line l that is perpendicular to ℓ_i. The projection of ℓ_i on l is a single point. For path set $\mathcal{L}$, we can obtain a measure of all these points obtained from projection. We denote it as M_1. Now let ℓ_i be projected onto l with a certain probability that is denoted by $\mathcal{J}(\ell_i)$. $\mathcal{J}(\ell_i)$ can be considered as the probability for penetrators taking path ℓ_i to be detected. We can obtain another measure of these weighted points generated from probabilistic projection. We denote it as M_2. M_2 actually provides a "sum" of all the detection possibilities along all the paths. The quality λ can be obtained as $\lambda = M_2 / M_1$.

We first consider a simple case where λ-1 D-coverage is evaluated and sensors are always active. When the penetration angle (θ in Fig. 2) is very small, the penetrator's movement pattern can be approximated as straight lines with one direction. We further assume the paths are uniformly distributed along the source boundary S, and every path $\ell \in \mathcal{L}$ is taken with the equal probability. These conditions can be relaxed later.

Let H denote the set of points covered by sensors. We have

$$\mathcal{J}(\ell_i) = \begin{cases} 1, & \text{when } \ \ell \cap H \neq \emptyset; \\ 0, & \text{otherwise}. \end{cases} \quad (1)$$

In calculating the H value for $\mathcal{J}(\ell_i)$, a belt with width D should be "strip off" in the path direction. Fig. 4 illustrates an example to get the quality metric λ, where a belt with width D has been striped off already.

Now we relax our conditions. For a penetration that may take different discrete intrusion angles

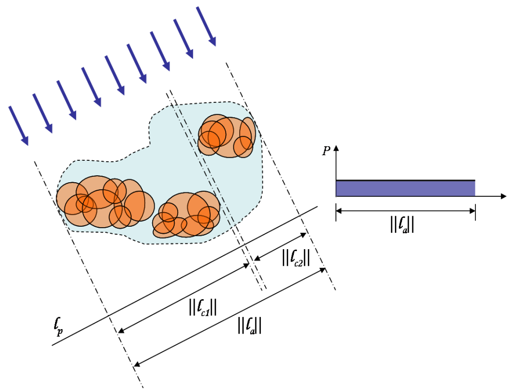

Figure 4. In the simple case, possible paths are uniformly distributed. They have a single direction. The possibility for each path to be taken is the same. We then have $\lambda = ||l_{c1}|| + ||l_{c2}||/||l_a||$.

$\theta_1, \theta_2, \cdots, \theta_n$ with different possibilities $P[\theta_1]$, $P[\theta_2]$, $\cdots$, $P[\theta_n]$, we have $\lambda = \sum_{i=1}^{n} \lambda(\theta_i) P[\theta_i]$, where $\lambda(\theta_i)$ is the quality over one specific direction. For an intrusion that may take different intrusion angles among some continuous range from θ_1 to θ_2 with probability distribution f, then $\lambda = \int_{\theta_1}^{\theta_2} \lambda(\theta) f(\theta) d\theta$, where $\lambda(\theta)$ is the quality of one specific direction. In one direction, if each path $\ell_i \in \mathcal{L}$ is chosen with probability $P[\ell_i]$, then $P[\ell_i]\mathcal{J}(\ell_i)$ is used.

This approach can also accommodate to the dynamic features of sensor nodes. We consider each sensor to be alive with probability p_l. Furthermore, to save energy, it is also natural to put sensors into duty cycles. In randomized independent scheduling, sensors independently follow the time cycle. At the beginning of periods, each sensor independently decides whether to remain active for this period with probability p_a or go to sleep with probability $1 - p_a$. Our approach can accommodate these dynamics. Suppose the path ℓ_i intersects the sensing range of one sensor, s_j, with length $l^i(s_j)$, then the probability for this sensor to detect the penetrator along ℓ_i is $p_d(\ell_i, s_j) = 1 - (1 - p_a p_l)^{\frac{l^i(s_j)}{D}}$. Then if a path ℓ_i intersects the sensing ranges of k sensors, $s_1, s_2, \cdots, s_k$, with length $l^i(s_1), \cdots, l^i(s_k)$, we obtain

$$\mathcal{J}(\ell_i) = 1 - \prod_{j=1}^{k} (1 - p_a p_l)^{\frac{l^i(s_j)}{D}}. \tag{2}$$

For concurrent K (K>1) D-coverage, the definition for $\mathcal{J}(\ell_i)$ is similar to those mentioned before. Thus the computation for λ is also similar. The only change is that H now denotes the area covered by K sensors. For sequential K (K>1) D-coverage, let $C(\ell_i)$ denote the set of sensors that intersect the path ℓ_i. Let $C(\ell_i, n)$ be a set of n sensors (ids) chosen from $C(\ell_i)$. Then we can obtain the detection probability by this set of sensors as

$$\prod_{x \in C(\ell_i, n)} p_d(\ell_i, s_x) \prod_{y \in C(\ell_i) - C(\ell_i, n)} (1 - p_d(\ell_i, s_y)), \tag{3}$$

To obtain $\mathcal{J}(\ell_i)$, we consider all possible $C(\ell_i, n)$ with $n \geq K$ and sum the detection probability from them together.

3.3. MC Based Comprehensive Approach

In this section, we introduce our comprehensive approach to evaluate the quality of D-coverage (λ) where we use Markov chain to approximate the real movement pattern of penetrators. Markov chain is a desired foundation of the solution for the important quality evaluation issue since it satisfies all the requirements aforementioned well.

3.3.1. Design Rationale. A Markov chain (MC) is a sequence $X_1, X_2, X_n \ldots$ of random variables and is characterized by the conditional distribution P($X_{n+1}|X_n$) called the transition probability of the process. We assume X_i takes value from a subset Ω of R^2, which means the number of states is infinite such that one penetration path ℓ can be modeled as a continuous-state chain. The Markovian property guarantees that at every point x in Ω there is a probability density function f_x of transition to all states in Ω. Based on this continuous modeling we will reach a theoretical description of a geographical region with distinctive properties encoded in *transition density functions* that is decided by the path set $\mathcal{L}$. Albeit its nicety, the continuous-state Markov chain is not easy to handle, so we proceed with a discretized version of it.

Consider a chain with several disjoint small sets A_i (for the definition of small sets refer to [10]). We could then define renewal time τ_n as the visiting time to one of the small sets. Then for a continuous-state Markov chain $\theta^{(t)}$, we have (where $\mathbf{I}$ is an indicator function) $\eta^{(t)} = \sum_{i=1}^{k} i \mathbf{I}_{A_i} \theta^{(t)}$ which is *not* a Markov chain; however the following subchain is $\xi^{(n)} = \eta^{(\tau_n)}$. We thus have the theorem as below [10].

Theorem 3.1: For a Harris-recurrent Markov chain $\theta^{(t)}$, the sequence $\xi^{(n)}$, which represents the successive indices of small sets visited by the chain $\theta^{(t)}$, is a homogeneous Markov chain on the finite state space $\{1, 2, 3, \ldots, k\}$.

Theorem 3.3.1 provides the basis for conducting region discretization in that it guarantees the Markovian property of the resulting discrete and usually computationally feasible chain. We can divide the area into multiple small convex areas conservatively with the diameter D. Fig. 5 shows an example. We are ready to construct a discrete time Markov chain model $\{X_i | i \geq 1\}$ for a penetration detection sensor network.

It can be represented in terms of an ensemble $(\mathcal{S}, \mathcal{I}, \mathcal{P})$, where $\mathcal{S}$ is the state space, $\mathcal{I}$ is the initial distribution and $\mathcal{P}$ is the transition matrix.

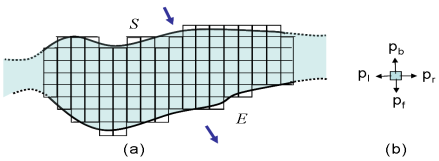

Figure 5. An area is divided into small rectangular areas. Probabilities for penetrators from one area to the adjacent are decided by their movement patterns.

3.3.2. Details. Each small area (rectangle in Fig. 5, but not necessary) is modeled as a transient state in $\mathcal{S}$. The transition only happens between adjacent areas, with probability decided by penetrator's movement pattern. These transient states are represented as $\{ts_1, ts_2, \cdots, ts_n\}$. In a typical case, we let the transition probabilities between adjacent areas be p_f, p_l, p_r and p_b respectively corresponding to moving forward, leftward, rightward and backward.

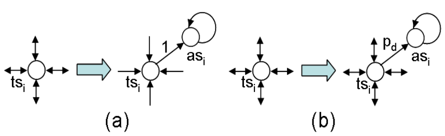

Figure 6. (a) If ts_i is covered by any sensor, then a new absorbing state as_i is added. The transition probability from ts_i to as_i is 1 if the penetrator at ts_i can be always detected. (b) If ts_i is covered by two sensors. Assume each sensor is being active with probability p. We have a new state transition diagram shown in the right with $p_d(ts_i) = 1 - (1-p)^2$.

Two categories of absorbing states are added into state space $\mathcal{S}$. The first category stems from sensor coverage. Suppose the center of a small polygon corresponding to ts_i is covered by some sensors. We then add a new absorbing state as_i with the transition probability $p_d(ts_i)$ from ts_i to as_i. When an penetrator moves into these absorbing states, it is considered detected. $p_d(ts_i)$ represents the probability for the penetrator to be detected at location ts_i. The reason why we model detections as *absorbing* states is that the penetrator will logically stay at these places forever from the moment when they are detected. In the case of λ-1 D-coverage, we do not need to consider its further movement. If sensors are always active, $p_d(ts_i) = 1$ and all other transitions going out from ts_i will be removed after as_i is added, as shown in Fig. 6(a). If we consider each sensor to be alive with probability p_l. And sensors are set into low duty circles, in which they are active for this period with probability p_a or go to sleep with probability $1 - p_a$. Assume that the center of a small area ts_i is covered by m sensors that can be represented as a set $C(ts_i)$ with elements being sensor ids, $C(ts_i) = \{id_1, id_2, \cdots, id_k\}$. We then have $p_d(ts_i) = 1 - (1 - p_a p_l)^{|C(ts_i)|}$. Fig. 6(b) shows an example. After we add $p_d(ts_i)$, the original probabilities for transitions going out from ts_i should be modified by the factor $1 - p_d(ts_i)$. Another category contains only one absorbing state, denoted as as^{out}. This absorbing state is added behind the destination boundary E as illustrated in Fig. 7. It does not indicate a detection. Instead, the penetrator entering this absorbing state implies it has not been detected when it crosses the surveillance area. It is reasonable to assume that once the penetrator has crossed the surveillance, it will not reenter it again. Now we have a complete state space and the transition probabilities.

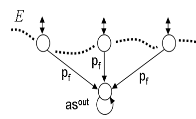

Figure 7. A new absorbing state as^{out} is added behind the destination boundary E.

Let $p_{in}(ts_i)$ denote the probability for the penetrator to start at an area ts_i that is at source boundary S. The initial probability distribution for the penetrator to enter the area can be obtained as $\mathcal{I} = \{p_{in}(ts_i)\}$.

Currently we are considering λ-1 D-coverage, an penetrator is discovered if it has been detected by at least one sensor. More specifically, the probability for the penetrator to be absorbed by as_i is the probability for this sensor network to detect the penetrator that has never been detected before. Hence, the absorbing probability for state as^{out}, the state behind the E, shows the probability for an penetrator having crossed the area but never been detected by any sensor. We denote this probability as P_{out}. The quality can be obtained as $\lambda = 1 - P_{out}$. The procedure to compute P_{out} is standard [11].

3.3.3. For λ-K (K>1) D-Coverage. The above basic Markov chain approach can be extended to evaluate λ-K (K>1) D-Coverage.

Concurrent λ-K (K>1) D-Coverage: The quality can still be obtained as $\lambda = 1 - P_{out}$. The procedure to compute P_{out} is similar to the above. There are two difference. First, a transition state ts_i has a absorbing state as_i associated with it only if it is covered by at least k sensors. Second, the transition probability from ts_i to as_i is changed to

$$\sum_{i=k}^{|C(ts_i)|} \binom{|C(ts_i)|}{k} {p_d}^i (1-p_d)^{|C(ts_i)|-i},$$

where $|C(ts_i)|$ is the number of sensors that cover ts_i.

Sequential λ-K (K>1) D-Coverage: From its definition, we have to tell which sensor contributes the detection each time. The new Markov chain should make this feasible.

Compared with the procedure at Subsection 3.3.2, the only difference in Markov chain construction here is that, we divide each absorbing state that stems from sensor coverage as_i into m absorbing sates $as_i^1, as_i^2, \cdots, as_i^m$. The value of m is given by $m = \binom{c}{1} + \binom{c}{2} + \cdots + \binom{c}{c}$ if $K > c$, or $m = \binom{c}{1} + \binom{c}{2} + \cdots + \binom{c}{K-1} + 1$ otherwise, where $c = |C(ts_i)|$. The transition probabilities from ts_i to each of the states $as_i^1, as_i^2, \cdots, as_i^m$ denotes the probability for the penetrator to be detected by a specific set of sensors in $C(ts_i)$ at location ts_i. We call ts_i their $location\ state$. Fig. 8(a) illustrates an example.

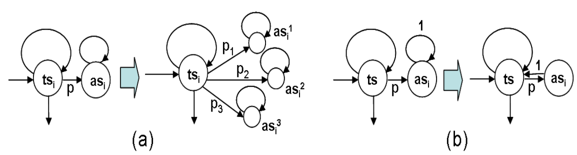

Figure 8. (a) Assume $C(ts_i) = \{a, b\}$, then $p_1 = p_d(1-p_d)$, $p_2 = p_d(1-p_d)$, $p_3 = p_d^2$. Entering as_i^1 implies the penetrator is detected only by sensor a. Entering as_i^2 implies it is detected only by sensor b. Entering as_i^3 implies the penetrator is detected by both sensors a and b. (b) as_i is changed to transition probability 1 to its location state.

There will be no changes for those states $ts_i \in \mathcal{S}$ with $|C(ts_i)| \leq 1$. In this new Markov chain, suppose there are totally n absorbing states that are denoted as $\{as_1, as_2, \cdots, as_{n-1}, as_n\}$. Note the nth absorbing state as_n is actually the one we added behind E, as^{out}, while others stem from sensor coverage. The new transition matrix is denoted as $\mathcal{P}_o$. The initial probability distribution for penetrators to appear at the states along S does not change and is denoted as $\mathcal{I}_o$.

When computing λ, all possible combinations of K sensors that make the detection should be considered. It is inevitable since there always exist paths going through the locations of these sensors with non-zero probabilities (some probabilities may be very small though).

The quality can still be obtained as $\lambda = 1 - P_{out}$, where P_{out} denotes the possibility for an penetrator to cross the area from S to E without being finally discovered. Let $P'_{out}(d)$ denote the possibility for the penetrator to cross the area after it has been detected by d different sensors. We then have: $P_{out} = \sum_{d=0}^{K-1} P'_{out}(d)$.

The following *Procedure-1* illustrates how to compute P_{out}. A function named *Compute_Pout* is defined. To determine P_{out}, this function will be invoked recursively. In the pseudocode, $\mathcal{P}_o$ is the initial transition matrix of the newly constructed Markov chain. $\mathcal{D}$ denotes the set of sensors that have detected the penetrator. At step 2, there is one matrix operation on $\mathcal{P}$ to calculate out absorbing probabilities into each absorbing state as_i, denoted by $P^{ab}[as_i]$, for $1 \leq i \leq n$; At step 5, we obtain the total number of different sensors which have detected the penetrator. If this number is larger than K, the penetrator has been discovered. Otherwise, the penetrator's further movement should be considered. At step 7 , we are to eliminate the further consideration of detection made by sensors that are already in $\mathcal{D}$. One way is to add the transition probability 1 from those as_is that reflect detection by sensors in $\mathcal{D}$ to their location state as illustrated in Fig. 8(b), and modify the transition probability to any other as_is that reflect detection by sensors some of which are in $\mathcal{D}$. Another way is to construct a new MC with the sensors in D are removed. At step 8, the new initial distribution is, probability $P^{ab}(as_i)$ for as_i's *location state* and 0 for other transient states.

Procedure-1: Pseudocode to compute P_{out}

```
     P_out = 0;  P = P_0;  I = I_o;  D = ∅;
1 :  Compute_Pout (P, I, D)
     begin
2 :  from P and I calculate out absorbing probabilities into
        each absorbing state as_i, P^ab(as_i), for 1 ≤ i ≤ n;
3 :  P_out = P_out + P^ab(as_n);
4 :  for i = 1 to n − 1
5 :      D' = D ∪ C(as_i);
6 :      if |D'| < K
7 :          get a new MC with transition probability P'
8 :          create a new initial distribution I';
9 :          Compute_Pout (P', I', D');
10 :     endif
11 :  endfor
     end
```

The key issue here is that the penetrator will logically stay at the state where it is detected. Modification at step 7 implies the detection made by those sensors that have already detected the penetrator will not be counted any more. If the penetrator has not been detected by K different sensors, its further movement that starts from the location where it is staying will be further considered. By the new distribution $\mathcal{I}'$ at step 8, the further movement that starts from a certain state can be specifically studied.

In above, Markov chain based approach is proposed to evaluate the quality of K directed surveillance, for general $K \geq 1$. However, there could be two concerns. 1) In many cases, the penetrator's movement pattern may not be known a priori. Although we can make some assumptions on it, it may not be accurate and hence the quality of directed surveillance evaluated may be affected; 2) The number of states may be too large to be handled. We will discuss these issues in the following sections.

3.3.4. Self Learning on Movement Pattern. In practice it is not always true that the penetrator's movement pattern has been exactly known before evaluation. Even when the pattern has been known, it may change with time or locations during penetrating. In this section, we present a solution to self-learn the probabilities for penetrators moving from one location to its adjacent locations in the surveillance network.

We exploit Hidden Markov Model (HMM). Once the penetrator enters the surveillance area, a sequence of tokens (messages) with time stamps indicating detection can be generated. This token sequence is from sensors along the intrusion path and naturally contains the information of transition probabilities between adjacent locations. By studying these token sequences, transition probabilities can be gradually updated close to their real values. In our scenario, "hidden" states are locations, and observations are token sequences. After the HMM is constructed, the Baum-Welch Expectation Maximization (EM) algorithm [13] can be used to update the initially assumed transition probabilities.

The HMM we construct can be represented as an ensemble $\delta = (\mathcal{S}_h, \Lambda, \mathcal{I}_h, \mathcal{A}, \mathcal{B})$ which is characterized by the following. $\mathcal{S}_h$ is the state space in the model, i.e., the set of small polygons. Λ, is the set of possible observations (tokens), $\Lambda = \{\tau_0, \tau_1, \tau_2, \cdots, \tau_k, \tau_{1,2} \cdots\}$. Here, τ_i denotes the token signaled to indicate an penetrator detection by sensor i. If sensors i and j signal the detection at the same time, their tokens are denoted as $\tau_{i,j}$. $\mathcal{I}_h$ is the initial state distribution that equals to $\mathcal{I}$ in aforementioned Markov model. $\mathcal{A}$ is the state transition probabilities we originally assumed and is $\mathcal{A} = \{a_{i,j}\}$. It needs to be modified after self learning from observation sequences. $\mathcal{B}$ is the probability distributions of tokens to be generated at states. As Fig. 9 shows, at each state some tokens can be generated, which is denoted as a set $\{\tau_X\}$. There is a probability associated with a specific token for its generation. Then $\mathcal{B} = \{b_i(\tau_X)\}$, where $b_i(\tau_X)$ denotes

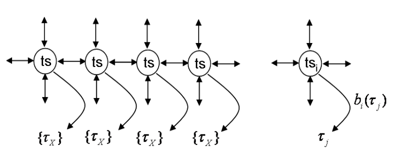

Figure 9. HMM for D-coverage. Token may be generated with some probability along the path. If state S_i is covered by sensor j, then it may generate τ_j with probability $b_i(\tau_j)$ when penetrators arrive at this state.

the probability for token τ_X to be generated at State i. For example, if State $S \in \mathcal{S}'$ and $C(S) = \{i, j\}$, assume each sensor is active with probability p_d we then have $b_S(\tau_i) = b_S(\tau_j) = p_d(1 - p_d)$, $b_S(\tau_{i,j}) = p_d^2$, and $b_S(\tau_0) = (1 - p_d)^2$.

In this HMM, locations are modeled as hidden states and the transition probabilities between adjacent locations follow some fixed but unknown distribution. As the penetrator continuously moves, a series of observations can be obtained, for example, $\tau_0\tau_{1,2}\tau_2\tau_2\tau_{2,3}\tau_4\tau_0\tau_0$ $\tau_0\tau_0 \cdots$. Here, τ_0 is a "null" token added if no token is generated by any sensor in a certain period. Empirically, if there are a large amount of consecutive τ_0's in the observation sequence, the penetrator can be considered to have left the surveillance area.

Let there be Q observation sequences, denoted by the set $O = \{O^{(1)}, O^{(2)}, \cdots, O^{(Q)}\}$, where $O^{(i)}$ is the ith observation sequence. Let o_t denote the observation at time t, $o_t \in \Lambda$, and $O^{(i)}$ can be denoted as $\{o_1^{(i)} o_2^{(i)} \cdots \; o_{T_i}^{(i)}\}$, where T_i is the time for the last token. Our objective can be now formalized as to maximize $P(O|\delta)$, that is, after constructing the HHM δ, according to the token sequences, we want to adjust $\mathcal{A}$ to $\bar{\mathcal{A}}$, where $\bar{\mathcal{A}} = \{\bar{a}_{i,j}\}$, such that the probability for generating those token sequences is maximized. $\bar{\mathcal{A}}$ will be much closer to the real transition probabilities than $\mathcal{A}$. The Baum-Welch Expectation Maximization (EM) algorithm [13] can be used here to get $\{\bar{a}_{i,j}\}$ to maximize $P(O|\delta)$. We comment that Baum-Welch EM algorithm is not the only one that can be used for our HMM based self-learning.

3.3.5. Complexity Reduction. The complexity of MC based approach can be reduced by two major ways as follows.

Divide-and-Conquer: Surveillance areas typically are of belt-like shape [15] and can be very long [1]. The area for the penetrator that may appear when it crosses the surveillance areas is often limited. It implies two locations that are vertically far from each other will have a very low probability to be on one path. Then we can divide the whole belt into multiple segments that are easier to handle, and compute P_{out} for each, and then combine them. We call this approach as Divide-and-Conquer.

Fig. 10 illustrates the method. We are to obtain the area where the expected visit time for each state is affected by edge effect, that is, to obtain a and c in Fig. 10. More specifically, edge effect implies that the penetrators at the left side of AC will enter somewhere within the area $ABCD$. Since EF delineates the place where the side effect almost ends, it implies the penetrator that starts on the left side of point A at source boundary S is almost impossible to reach the right side of EF. Then we can use a conservatively small ϵ to get a in terms of state number n_a,$n_a \geq \log_{p_r} \epsilon$. Similarly, we have n_c by considering p_l.

With this method, the computation complexity can be greatly reduced. Suppose originally there are totally m transient states, the time complexity to get absorbing

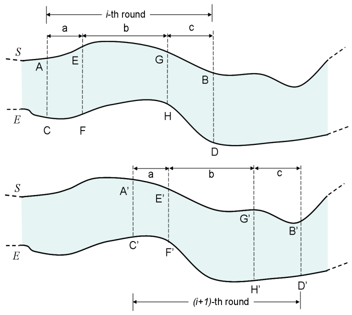

Figure 10. In the $i-$th round we compute the expected visiting time for states in area $ABCD$, but only those results for states within area $EFGH$ are considered valid. In the $(i+1)-$th round, we compute the expected visiting time for states in area $A'B'C'D'$, the valid area is $E'F'G'H'$.

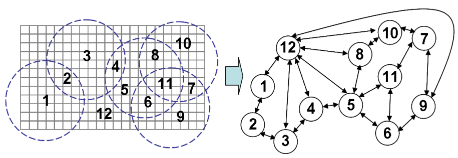

Figure 11. There are 299 transient states and 381 absorbing states before combination. After combination, we have 12 transient and 27 absorbing states.

probability is $\Theta(m^2)$. If it has been divided into α segments, the time complexity is then $\Theta((\frac{m}{\alpha})^2)$.

State Combination: First considering those transient states in $\mathcal{S}$ with nonempty coverage set, we take all those that are covered by same sensors and unite them into a single set. This set becomes a new transient state in $\mathcal{S}'$. Assuming this new state is ts'_i, we define $C(ts'_i) = C(ts)$ for a transient state $ts \in ts'_i$. Then we let absorbing states that are originally associated with ts now associate with ts'_i. We repeat this procedure until all the transient states in $\mathcal{S}$ with nonempty coverage set are handled. We then combine those transient states with empty coverage set into a single set, which is also a new transient state in $\mathcal{S}'$. There is no absorbing state associated with this new state. Finally, we put a set containing the only absorbing state behind the destination boundary E, as^{out}, into the the $\mathcal{S}'$.

This method is inspired by [18], which is for performance evaluation in real-life industrial systems. The example in Fig. 11 illustrates its effectiveness.

We define $\{X'_i | i \geq 1\}$ in the new state space as

$$X'_i = \begin{cases} ts'_i, & \text{if } X_n = ts \text{ and } ts \in ts'_i \\ \text{undefined}, & \text{otherwise} \end{cases} \quad (4)$$

Proposition 3.1: $\{X'_i | i \geq 1\}$ is still a Markov chain.

Proof: We first proof $\{X'_i | i \geq 1\}$ defined is always a stochastic process, since in general X'_i defined in (4) is not even a function. Notice in $\mathcal{S}'$ there is at least one positive recurrent state S'_a. Then each infinite path has infinitely many occurrences of state S'_a or other absorbing states with probability one. Hence, X'_i is a complete $\mathcal{S}'$-valued function. We further have to show that each X'_i is a random variable. This follows easily from the fact that each set of infinite paths is measurable. Now we are to prove the Markov property holds for $\{X'_i | i \geq 1\}$. Markov property holds for $\{X_i | i \geq 1\}$, i.e., we have $\{X_{n+1} = S_{n+1} | X_n = S_n, \cdots, X_1 = S_1\} = \{X_{n+1} = S_{n+1} | X_n = S_n\}$. Now we rewrite S_i for any i as S'_j if $S_i \in S'_j$. Sine the construction procedure is complete and there only exits unique one to one $\in$ relationship between a state in $\mathcal{S}$ and a state in $\mathcal{S}'$, we have $\{X'_{n+1} = S'_{n+1} | X'_n = S'_n, \cdots, X'_1 = S'_1\} = \{X'_{n+1} = S'_{n+1} | X'_n = S'_n\}$. $\square$

Furthermore, we define new transition probabilities $a'_{i,j}$ from a new state ts'_i to another new state ts'_j as follows. Let $\mathcal{N}^v(ts_i))$ denote the expected visited number for a transient state $S_i \in \mathcal{S}$. Assume that ts'_j is a transient state, and there are n_i states $ts_i{}^1, ts_i{}^2, \cdots, ts_i{}^{n_i}$ in ts'_i and n_j states $ts_j{}^1, ts_j{}^2, \cdots, ts_j{}^{n_j}$ in ts'_j. We then assign the transition probability between transient states as

$$a'_{i,j} = \frac{\sum_p^{n_i} (\sum_q^{n_j} \mathcal{P}(S_i{}^p, S_j{}^q) \mathcal{N}^v(S_i{}^p))}{\sum_p^{n_i} \mathcal{N}^v(S_i{}^p)}. \quad (5)$$

The self transition probability for S'_i is then defined as $a'_{i,i} = 1 - \sum_{j \neq i} a'_{i,j}$. We also define the new initial distribution $\mathcal{I}'$ as $P_{in}(ts'_i) = \sum_{ts \in S'_i} P_{in}(ts)$.

Proposition 3.2: The λ obtained from $\{X'_i | i \geq 1\}$ is the same as that from $\{X_i | i \geq 1\}$.

Proof: Assume $S'_i, S'_j \in \mathcal{S}'$, and $S_i{}^1, S_i{}^2, \cdots, S_i{}^{n_i} \in S'_i$ and $S_j{}^1, S_j{}^2, \cdots, S_j{}^{n_j} \in S'_j$.

$$\begin{aligned} & \mathcal{P}(X'_{t+1} = S'_j | X'_t = S'_i) = \frac{\mathcal{P}(X'_{t+1} = S'_j, X'_t = S'_i)}{\mathcal{P}(X'_t = S'_i)} \\ = & \frac{\sum_{p=1}^{n_i} \mathcal{P}(X'_{t+1} = S'_j, X'_t = S'_i | X'_t = S_i{}^p) \mathcal{P}(X'_t = S_i{}^p)}{\mathcal{P}(X'_t = S'_i)} \\ = & \frac{\sum_{p=1}^{n_i} \mathcal{P}(X'_{t+1} = S'_j | X'_t = S_i{}^p) \mathcal{P}(X'_t = S_i{}^p)}{\mathcal{P}(X'_t = S'_i)}. \\ \rightarrow & \mathcal{P}(X'_t = S'_i) \\ = & \frac{\sum_{p=1}^{n_i} \mathcal{P}(X'_{t+1} = S'_j | X'_t = S_i{}^p) \mathcal{P}(X'_t = S_i{}^p)}{\mathcal{P}(X'_{t+1} = S'_j | X'_t = S'_i)} \\ = & \frac{\sum_{q=1}^{n_j} (\sum_{p=1}^{n_i} \mathcal{P}(X'_{t+1} = S_j{}^q | X'_t = S_i{}^p) \mathcal{P}(X'_t = S_i{}^p))}{\mathcal{P}(X'_{t+1} = S'_j | X'_t = S'_i)}. \end{aligned}$$

We replace $\mathcal{P}(X'_{t+1} = S'_j | X'_t = S'_i)$ above by

$$\mathcal{P}(X'_{t+1} = S'_j | X'_t = S'_i) = \frac{\sum_q^{n_j} (\sum_p^{n_i} \mathcal{P}(S_i{}^p, S_j{}^q) \mathcal{N}^v(S_i{}^p))}{\sum_p^{n_i} \mathcal{N}^v(S_i{}^p)}.$$

Then take $\sum_{t=0}^{\infty}$ on both sides, then we have, $N^v(S'_i) = \sum_p^{n_i} \mathcal{N}^v(S_i{}^p) = \sum_{S \in S'_i} N^v(S)$. It implies that in our new chain, the expected visiting time at each sensor's sensing range is the same as that in the original chain. Then we have,

$$
\begin{aligned}
A(S_i') &= N^v(S_i')\frac{\sum_q^{n_a}(\sum_p^{n_i}\mathcal{P}(S_i{}^p, S_a{}^q)\mathcal{N}^v(S_i{}^p))}{\sum_p^{n_i}\mathcal{N}^v(S_i{}^p)} \\
&= \sum_q^{n_a}(\sum_p^{n_i}\mathcal{P}(S_i{}^p, S_a{}^q)\mathcal{N}^v(S_i{}^p)) \\
&= \sum_{S\in S_i'} A(S).
\end{aligned}
$$

It implies that the absorbing probability at each sensor's sensing range in the new chain is the same as that in the original chain. Hence, the λ calculated out by new chain is exactly the same that calculated in the original chain. □

3.4. Discussions

The projection based approach approximates the penetrator's movements by straight lines and uses projections to transfer a two-dimensional geometry problem to a one-dimensional one. It is simple and appropriate for cases where the penetrator has a strong intention to penetrate to a specific destination. The Markov chain based approach on the other hand can incorporate more complicated penetrating movement patterns. Markov chain is a well studied topic. There have been many existing results and methods which can be directly applied to an approach based on it. Hidden Markov Model (HMM) based self learning is an example. Using above HMM based approach, not only can we learn the penetrator movement pattern that was unknown, but can capture unexpected transition probabilities between locations, e.g., those that result from geographical characteristics, at different locations in the surveillance area.

Divide-and-conquer and state combination can be combined. They can also benefit HHM based self-learning by making it more efficient. Since transition exists only locally, the transition matrixes are always very sparse. Techniques used for sparse matrix operations can be further applied to reduce computation complexity [17]. Further optimization of computation procedure is out of the interest of this paper.

Note that our approaches are not a panacea for all cases. Our Markov chain based approach can not handle the case where the penetrator has a certain degree of memory and its movement is not independent from previous steps. Addressing this problem is a part of our future research.

4. Evaluations and Observations

On Deterministic Deployment: We use three surveillance shapes to evaluate the accuracy of our evaluation approaches, i.e. the simple projection based approach and the comprehensive Markov chain based approach. The three topologies as shown in Fig. 12, Fig. 13, and Fig. 14 can be considered as abstraction from the three applications described in Section 2. Sensors are homogenous with sensing radius $4m$. The location of sensors for these instances are randomly generated. Fig. 15, Fig. 16, and Fig. 17 shows the quality λ of D-coverage for above three instances while detection condition D increases. D decides the length of the grids' diagonal. Due to the strong practical meaning of λ, results from simulations can be directly compared with those obtained from simple projection approach and comprehensive Markov chain based approach. The quality for λ-1 and λ-2 D-coverage are considered. Penetrator movement model parameters are of $p_f = 0.6$, $p_b = 0.1$, $p_l = 0.15$ and $p_r = 0.15$. Sensors are scheduled to be active with probability 0.9. Simulation results are the average from 1000 runs. The fidelity between simulation and analysis data demonstrates our evaluation approaches can accurately present the quality of directed coverage. Furthermore, we notice as the detection condition D increases, the quality of D-coverage degrades.

On Random Deployment: In this subsection, we study the quality of D-coverage in independent and uniform sensor deployments. We show the results presented in [15] as a special case of ours when the grid size is very small. Focusing on absolute barrier coverage, [15] gives the sufficient condition for all orthogonal crossing lines to be 1-covered with high probability. It defines c_{high} as $c_{high} = \min_n\{c(s) : c(s) \geq 1 + \phi(np)/\log(np)\}$, where $\phi(np)$ is suggested to be $\sqrt{\log\log(np)}$, p is the active probability and n is the sensor number. Theorem 6.1 in [15] states that if $2npr/(s\log(np)) \geq c_{high}$, then the probability for all orthogonal crossing lines to be 1-covered should be close to 1. Here, r is the radius for sensing disk and s is a measure of belt shape of barrier coverage.

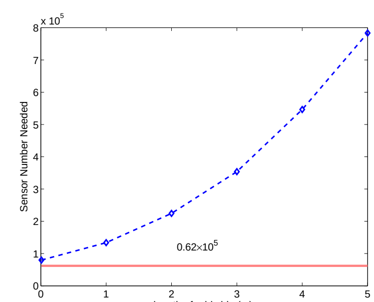

Figure 18. The horizontal line is the sensor number needed to achieve absolute barrier coverage obtained from [15]. The diamonds represent the data from our Markov chain based evaluation approach.

We change the grid size from $0.01m \times 0.01m$ to $5m \times 5m$. We then consider the same configuration for the sensor network as in [15]. We study a $100m \times 10km$ belt area, where s equals 10. Sensor range is a disk with

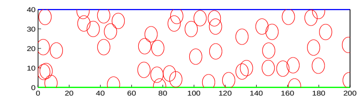

Figure 12. A belt area with area $200m \times 40m$ on which 50 sensors are deployed. S is the bottom boundary. E is the upper boundary.

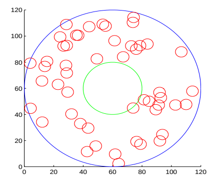

Figure 13. A ring area with width $40m$ where 50 sensors are deployed. S is the inner circle. E is the outer circle.

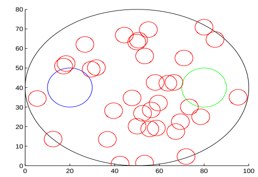

Figure 14. An ellipse area with long axis $100m$ and short axis $80m$ with 35 sensors deployed. S is the right inner circle. E is the left one.

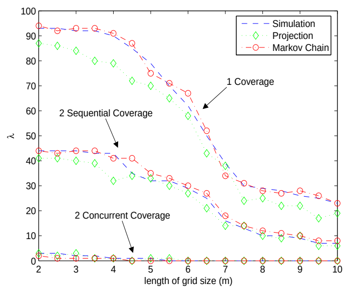

Figure 15. Quality for Fig.12.

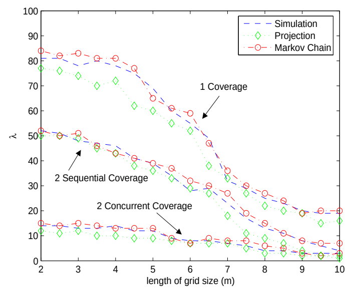

Figure 16. Quality for Fig.13.

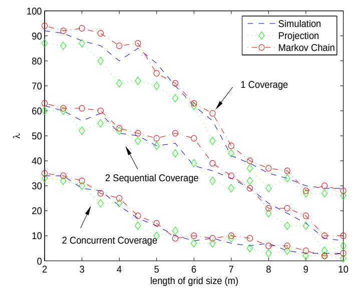

Figure 17. Quality for Fig.14.

radius $10m$, then $r = 0.01$ after normalization. Active probability $p_d = 0.1$. In Fig. 18, for each penetrator speed, we first use 100 randomly deployments with certain number of sensors to calculate λ. The smallest sensor number is then taken if the average value of these λs is larger than 99.9%.

From Fig. 18, when the grid size is very small, i.e., $0.01m \times 0.01m$, our result is close to the weak sufficient condition presented in [15]. The critical condition in [15] provides a good reference when detection condition D is very small . But when detection condition D is large, it has room to enhance.

5. Conclusion

In this paper, we introduce a new concept of coverage in wireless sensor networks called D-Coverage. We also propose two evaluation approaches, i.e. the projection based simple approach and a more comprehensive Markov chain based approach. Performance data show that our evaluation approaches can accurately evaluate the quality of D-coverage and provide good guidelines for sensor network deployment and run-time repair.

References

[1] A. Arora, R. Ramnath, etc., EXCAL: Elements of an Extreme Scale Wireless Sensor Networks, *RTCSA*, 2005.

[2] B. Liu and D. Towsley, On the coverage and detectability of large-scale wireless sensor networks, *MASS*, 2003.

[3] C. Gui and P. Mohapatra, Power conservation and quality of surveillance in target tracking sensor networks, *ACM MobiCom*, 2004.

[4] A. Chen, T.H. Lai and D. Xuan, Measuring and Guaranteeing Quality of Barrier-Coverage in Wireless Sensor Networks, A *ACM MobiHoc*, 2008.

[5] P. Balister, Z. Zheng, S. Kumar, and P. Sinha, Trap Coverage: Allowing Coverage Holes of Bounded Diameter in Wireless Sensor Networks, *IEEE INFOCOM*, 2009.

[6] P. Balister, B. Bollobas, A. Sarkar, and S. Kumar, Reliable Density Estimates for Achieving Coverage and Connectivity in Thin Strips of Finite Length, *ACM MobiCom*, 2007.

[7] A. Chen, S. Kumar, and T. H. Lai, Designing Localized Algorithms for Barrier Coverage, *ACM Mobicom*, 2007.

[8] M. K. Watfa and S. Commuri, Power Conservation Approaches to the Border Coverage Problem in Wireless Sensor Networks, *ICWN*, 2006.

[9] B. Liu, O. Dousse, J. Wang et al., Strong barrier coverage of wireless sensor networks, *ACM MobiHoc*, 2008.

[10] C. G.-Jouyaux; C. P. Robert Discretization of Continuous Markov Chains and Markov Chain Monte Carlo Convergence Assessment, *Journal of the American Statistical Association, Vol. 93, No. 443., pp. 1055-1067*, 1988.

[11] G. F. Lawler, Introduction To Stochastic Processes, *Chapman Hall/CRC*, 1995.

[12] H. Zhang and J. Hou, On Deriving the Upper Bound of α-Lifetime for Large Sensor Networks, *ACM MobiHoc*, 2004.

[13] L. E. Baum, T. Peterie, G. Souled, and N. Weiss, A Maximization Technique Occurring in the Statistical Analysis of Probabilistic Functions of Markov Chains, *Ann. Math. Statist., vol. 41, no.1*, 1970.

[14] P. Hall, Introduction to the Theory of Coverage Processes, *John Wiley & Sons*, 1988.

[15] S. Kumar, T. H. Lai, and A. Arora, Barrier Coverage With Wireless Sensors, *ACM MobiCom* , 2005.

[16] S. Kumar, T. H. Lai, and J. Balogh, On k-coverage in a Mostly Sleeping Sensor Network, *ACM MobiCom*, 2004.

[17] S. Pissanetzky, Sparse Matrix Technology, *Academic Press, London*, 1984.

[18] Y. Pribadi, J.P.M. Voeten and B.D. Theelen, Reducing Markov chains for performance evaluation, *The 2nd Workshop on Embedded Systems*, 2001.