Instructions:

1) This test is 4 pages in length.

2) You have 75 minutes to complete and turn in this test.

3) Short-answer questions include a guideline for how many sentences to write. Respond in complete English sentences.

4) This test is closed books, notes, phones, smartwatches, laptops, friends, neighbors, etc.

5) Use the backs of pages in this test packet for scratch work. If you write more than a final answer in the area next to a question, circle your final answer.

6) Write and sign the following: “I pledge my Honor that I have not cheated, and will not cheat, on this test.”

_________________________________________________________  ________________
_______________________________________________________________________

Signed: ______________________________________________
1. [5 points]
   What are first-class functions? [1-2 sentences]

2. [10 points]
   a) Provide an example of an ML program that violates the value restriction.

   b) Rewrite your example from Part (a) into an equivalent program that does not violate the value restriction.

3. [25 points]
   a) Implement a function \texttt{filter} that takes (in curried form) a function $F$ and a list $L$ of triples. Function $F$ must take a triple and return a bool. Function \texttt{filter} returns a list containing only those triples in $L$ for which $F$ returns \texttt{true}. Use ML syntax in your implementation (including pattern matching, anonymous variables, and as-bindings when appropriate). Do not call any built-in higher-order functions (like \texttt{map}, \texttt{foldl}, or \texttt{foldr}) in your implementation.

   b) What type does \texttt{filter} have?
4. [20 points]
Consider the following function F.

   fun F g s = foldl (fn(x, y)=>(g(x) andalso y)) true s;

a) What is the type of F?

b) Succinctly summarize what function F does. (1 sentence)

c) What is the type of the expression \( F \ (\text{fn } x=>x<5) \ [2, 4, \sim6, \sim8, \sim3, 5, \sim6, \sim10] \)?

d) To what value does \( F \ (\text{fn } x=>x<5) \ [2, 4, \sim6, \sim8, \sim3, 5, \sim6, \sim10] \) evaluate?

e) What is the type of the expression \( F \ (\text{fn } x=>x) \)?

f) Implement the simplest possible function that is equivalent to F but that does not use a built-in function like foldl. Use ML syntax (including pattern matching, anonymous variables, and as-bindings when appropriate).
5. [40 points]
Recall the Z and S rules from class, used to define natural numbers. E.g., Z nat is derivable using the Z rule, and S(S(Z)) nat is derivable using the S rule twice and then the Z rule. For the remainder of this assignment, assume that every use of the symbol N identifies a natural number, so implicitly, N nat is derivable. Now let’s define the PZ and PS rules for addition as follows.

\[
\begin{align*}
N_1 + N_2 &= N_3 & \text{PZ} \\
N + Z &= N & \text{PS}
\end{align*}
\]

Using this deductive system, prove that addition is commutative.

*Theorem.* For all \( N_1, N_2, \) and \( N_3 \): if \( N_1 + N_2 = N_3 \) then \( N_2 + N_1 = N_3 \).