Objectives
1. To gain experience writing inference rules in deductive systems.
2. To practice proving properties of judgments by induction on their derivations.

Due Date: Tuesday, September 23, 2014 (at the beginning of class, 5:00 pm).

Assignment Description
Do the following by yourself.

(1) Define a deductive system having two judgment forms:
   a) Judgments of the form \( N \text{ Nat} \) are valid iff \( N \) is a natural number. Just use the same 2 inference rules we discussed in class.
   b) Judgments of the form \( N_1 \text{-} N_2 = N_3 \) are valid iff subtracting natural-number \( N_2 \) from natural-number \( N_1 \) produces natural-number \( N_3 \). For example, \( S(S(S(Z)))\text{-}S(S(Z))=S(Z) \) should be derivable, but for all \( N \), \( S(S(Z))\text{-}S(S(S(Z)))=N \) should not be derivable. Your rules for subtraction judgments can implicitly assume that all numbers involved are natural numbers; your rules for subtraction judgments therefore don’t have to contain judgments of the form \( N \text{ nat} \). (Essentially, we’re assuming that, in subtraction judgments, the symbol \( N \) always refers to a valid natural number.)

(2) Using your definitions from Step (1), formally prove the following Lemma A.
   Lemma A. \( \forall N: (N \text{ nat} \Rightarrow N \text{-} N=Z) \)

(3) [This step is for graduate students; undergrads may complete this step for +10% extra credit] Again using your definitions from Step (1), formally prove the following Lemma B.
   Lemma B. \( \forall N_1, N_2, N_3: (N_1 \text{-} S(N_2)=N_3 \Rightarrow N_1\text{-}N_2=S(N_3)) \)

(4) Using your definitions from Step (1), formally prove the following Theorem C.
   Theorem C. \( \forall N_1, N_2, N_3: (N_1\text{-}N_2=N_3 \Rightarrow N_1\text{-}N_3=N_2) \)
   If helpful, your proof of Theorem C can assume that Lemmas A and B hold.

Grading Notes
Partial credit is always possible. If you get stuck, just explain informally whatever ideas you’re having trouble stating formally.

Submission Notes
- Turn in a hardcopy (handwritten or printed) version of your solutions. Please do not email solutions or upload them into Canvas.
- Write the following pledge at the end of your submission: “I pledge my Honor that I have not cheated, and will not cheat, on this assignment.” Sign your name after the pledge. Not including this pledge will lower your grade 50%.
- You may submit solutions up to 2 days late (i.e., by 5pm on Thursday, September 25) with a 15% penalty.
- If you think there’s a chance you’ll be absent or late for class on the date this assignment is due, you’re welcome to submit solutions early by giving them to me or a TA before or after class, or during any of our office hours.