Assignment Description

Do the following by yourself (please don’t discuss solutions until after the due date).

1. (a) Read the attached handout on normalization.
(b) Prove the normalization property of the simply typed lambda calculus, as is done in the handout, but using two-column proofs. In other words, rework the handout’s proofs to be in a two-column format.

2. Consider a type-safe, CBV, simply typed lambda calculus $L$ having left-to-right evaluation. The base types in $L$ are $\text{nat}$ and $\text{float}$, with $\text{nat}$ a subtype of $\text{float}$. The expressions $e$ in $L$ are: $n, f, \sqrt{e}, \lceil e \rceil, \lambda x: \tau.e, x$, and $e(e')$. Here $n$ is a nat literal, $f$ is a float literal, $\sqrt{e}$ returns a float approximating the square root of the nat-type subexpression $e$, $\lceil e \rceil$ returns a nat for the ceiling of the float-type subexpression $e$, and the $\lambda x: \tau.e, x$, and $e(e')$ expressions have the usual meanings.
(a) Define $L$’s syntax and semantics (use evaluation contexts).
(b) State—without proof—all the lemmas/theorems/corollaries you’d use to prove that $L$ is type safe. (Hint: You’ll want to include a subtyping-inversion lemma.)
(c) Prove the preciseness of $L$’s subtyping relation (in a 2-column format).

3. [15% Extra credit] Supply the proofs for Problem 2b above.