CIS 4930/6930: Principles of Cyber-Physical Systems

Chapter 4: Hybrid Systems

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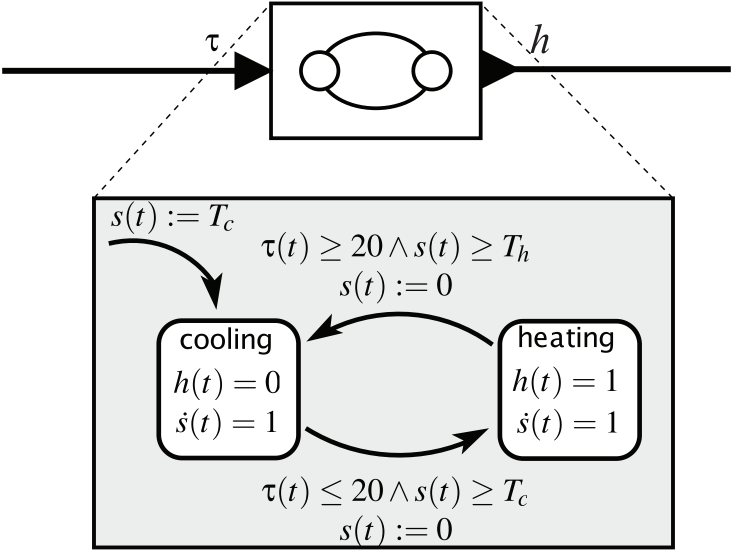
Hybrid Automata

Timed Automaton Model of a Thermostat

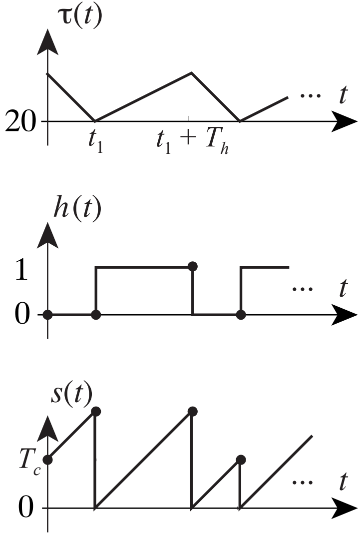


Possible Execution of the Timed Thermostat Model

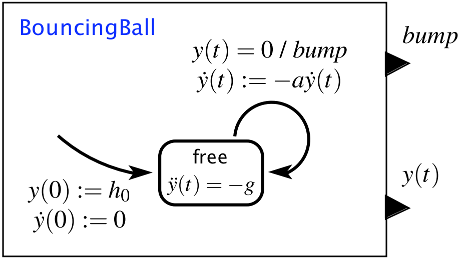
(a)

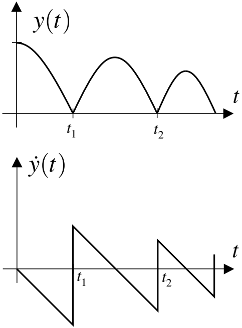
(b)

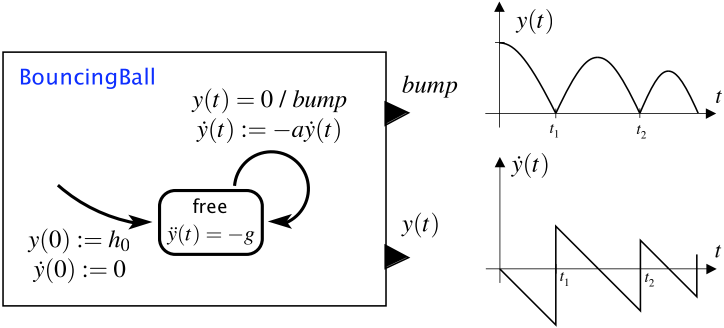
(c)



Higher Order Dynamics: Bouncing Ball

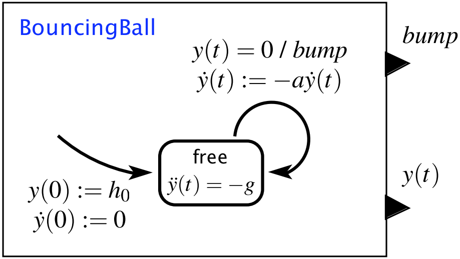


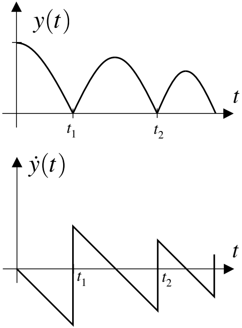




$$y(0) = h_0$$
$$\dot{y}(t) = -gt$$
$$y(t) = y(0) + \int_0^t \dot{y}(\tau) d\tau = h_0 - \frac{1}{2} g t^2$$

Higher Order Dynamics: Bouncing Ball

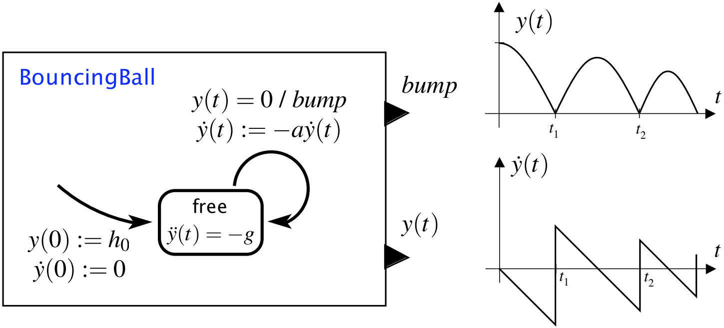




$$
\begin{aligned}
y(0) &= h_0 \\
\dot{y}(t) &= -gt \\
y(t) &= y(0) + \int_0^t \dot{y}(\tau)d\tau = h_0 - \frac{1}{2}gt^2
\end{aligned}
$$

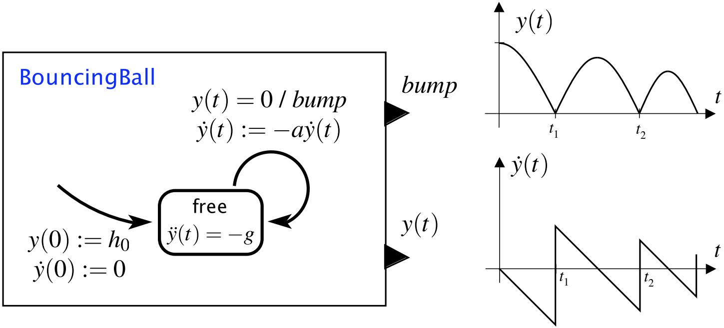
$$y(t_1) = 0, \quad h_0 - \frac{1}{2}gt_1^2 = 0, \text{ thus } t_1 = \sqrt{2h_0/g}$$

Higher Order Dynamics: Bouncing Ball



At t_1, $y(t)_1 = 0$. The *bump* transition takes place with new speed $-a\dot{y}(t_1)$.

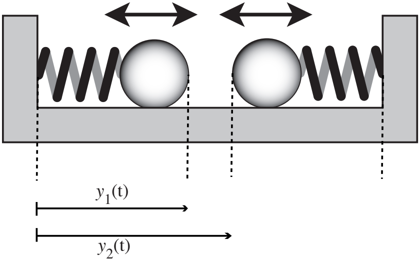
Higher Order Dynamics: Bouncing Ball

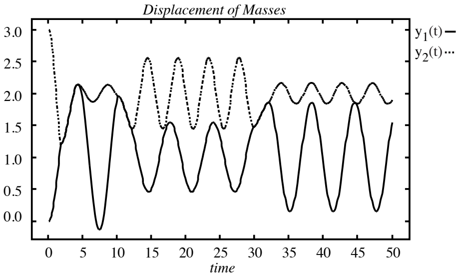


At t_1, $y(t)_1 = 0$. The *bump* transition takes place with new speed $-a\dot{y}(t_1)$.

$$\dot{y}(t) = -a\dot{y}(t_1) - gt \ \ (t > t_1)$$

Sticky Masses Example





Sticky Masses Example System Dynamics

- Let p_1 and p_2 be neutral places of the two springs.
  - The forces due to the springs are zero.
- Suppose the spring force is proportional to the displacement.
- When **apart**, forces due to the springs:

$$
\begin{aligned}
F_1 &= k_1(p_1 - y_1(t)) \\
F_2 &= k_2(p_2 - y_2(t))
\end{aligned}
$$

- Under Newton's 2nd Law (i.e., $F = ma$):

$$
\begin{aligned}
\ddot{y}_1(t) &= k_1(p_1 - y_1(t))/m_1 \\
\ddot{y}_2(t) &= k_2(p_2 - y_2(t))/m_2
\end{aligned}
$$

Sticky Masses Example System Dynamics

- When stuck **together**, pulled in opposite directions by two springs:

$$\begin{aligned} F &= F_1 + F_2 \\ m &= m_1 + m_2 \\ & y(t) = y_1(t) = y_2(t) \end{aligned}$$

$$\ddot{y}(t) = \frac{k_1 p_1 + k_2 p_2 - (k_1 + k_2) y(t)}{m_1 + m_2}$$

Sticky Masses Example System Dynamics

- Guard on the **apart** to **together** transition is: $y_1(t) = y_2(t)$.
- Initial velocity of combined mass, $\dot{y}(t)$, set by conservation of momentum:

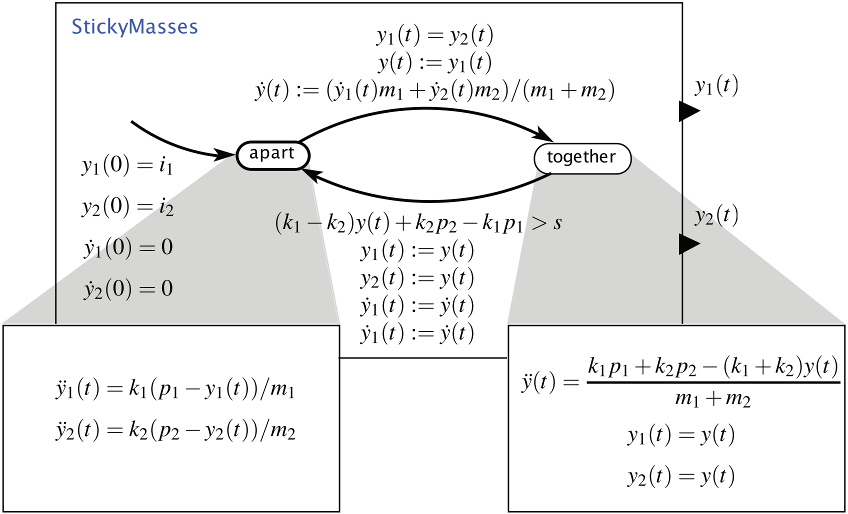
$$\begin{aligned} \dot{y}(t)(m_1 + m_2) &= \dot{y}_1(t)m_1 + \dot{y}_2(t)m_2 \\ \dot{y}(t) &= \frac{\dot{y}_1(t)m_1 + \dot{y}_2(t)m_2}{(m_1 + m_2)} \end{aligned}$$

- Guard on the **together** to **apart** transition is:

$$F_2 - F_1 = (k_1 - k_2)y(t) + k_2 p_2 - k_1 p_1 > s$$

  where s represents the stickiness of the two masses.
- This transition occurs when the right-pulling force, $k_2(p_2 - y(t))$, exceeds the left-pulling force, $k_1(p_1 - y(t))$, by the stickiness s.

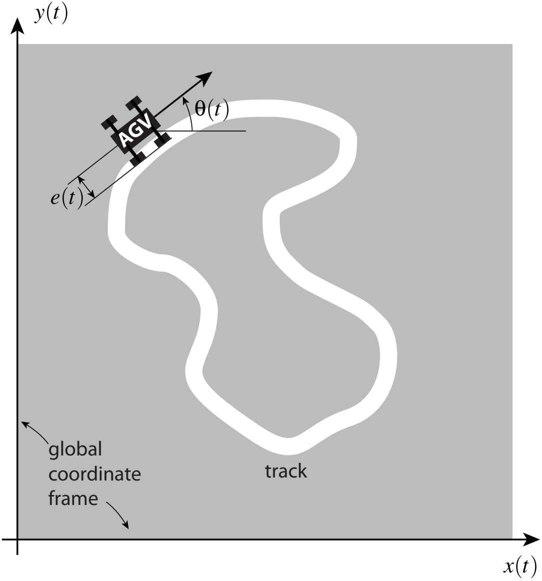
Hybrid System Model for Sticky Masses



Control Systems

- A **control system** includes:
  - The **plant** - the physical process that is to be controlled.
  - The environment.
  - The sensors.
  - The controller.
- The controller has two levels:
  - **Supervisory control** determines the mode transition structure.
  - **Low-level control** determines the time-based inputs to the plant.
- Supervisory controller determines the strategy while the low-level controller implements the strategy.
- Hybrid systems are ideal for modeling control systems.

Automated Guided Vehicle (AGV) Example



AGV Dynamics

- The speed is $u(t)$ is restricted to:

$$0 \leq u(t) \leq 10 \text{ mph}$$

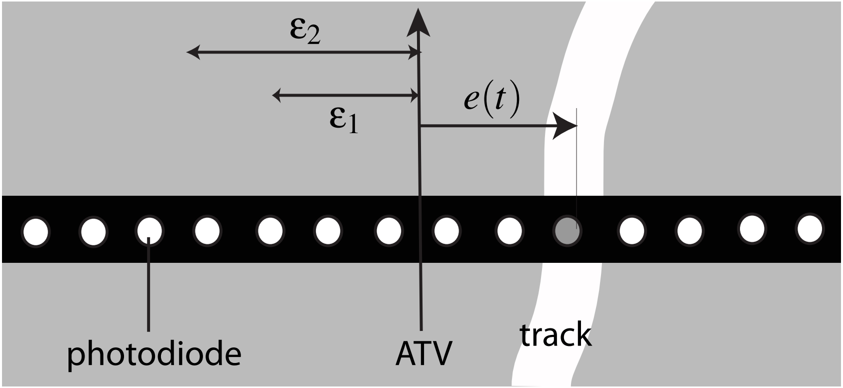
- The angular speed is $\omega(t)$ is restricted to:

$$-\pi \leq \omega(t) \leq \pi \text{ radians/second}$$

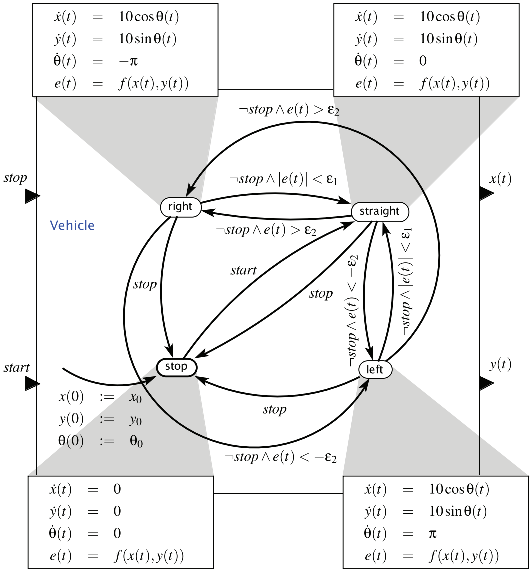
- Position is $(x(t), y(t)) \in \mathbb{R}^2$ and angle is $\theta(t) \in (-\pi, \pi]$.
- The motion of the AGV is defined by the differential equations:

$$\begin{aligned} \dot{x}(t) &= u(t)\, cos\, \theta(t) \\ \dot{y}(t) &= u(t)\, sin\, \theta(t) \\ \dot{\theta}(t) &= \omega(t) \end{aligned}$$

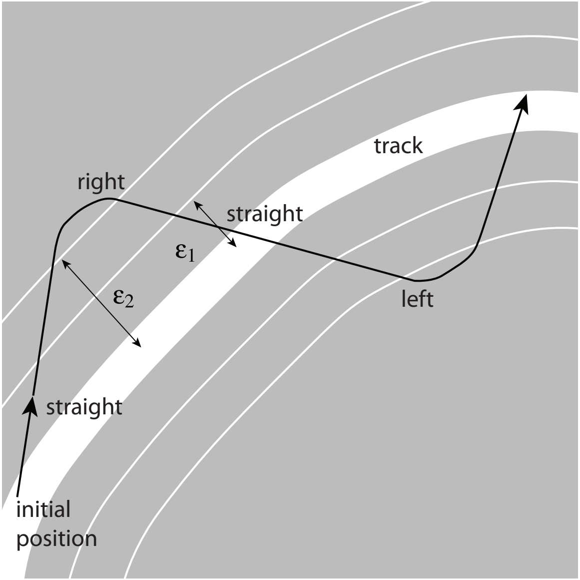
Determining the Error in Position



Hybrid System Model for the AGV Example



A Trajectory for the AGV Example



AGV Example Summary

- Plant is the differential equations governing the AGV motion.
- Environment is the closed track.
- Sensor is $e(t)$ which gives the AGV position relative to the track.
- Supervisory controller are the four modes and guards to switch b/w them.
- Low-level controller is the specification of inputs to the plant u and ω.

Concluding Remarks

- Hybrid systems are a bridge between state-based and time-based models which allow for the description of real-world systems.
- Discrete transitions are used to change the mode of operation.
- These transitions are taken when guards are satisfied that include both inputs and predicates on continuous variables.
- The change in mode may result in a change in continuous behavior.
- Analysis of hybrid systems is complicated by the fact that both state-based and time-based analysis is required.