# >>> Assignment #4 for Simulation (CAP 4800) <<<

## >>> SOLUTIONS <<<

This assignment covers material from the fourth week of class lecture.

**Problem #1** (35 points)

Determine *X*, *Ts*, *U*, *W*, *Wq*, *L*, and *Lq* for the following single-server queueing system for the time period 0 to 150 seconds. Carefully show your work including all pertinent figures and formulas. **Hint:** Review your week #4 reading (MacDougall, Chapter 1).

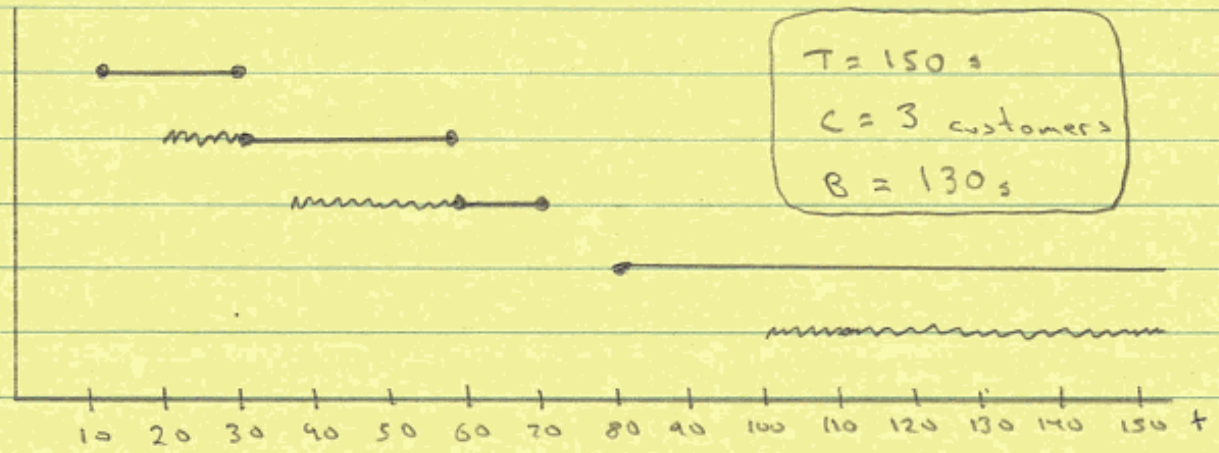
- Arrival #1 at time = 10 seconds with service time = 20 seconds
- Arrival #2 at time = 20 seconds with service time = 30 seconds
- Arrival #3 at time = 35 seconds with service time = 10 seconds
- Arrival #4 at time = 80 seconds with service time = 120 seconds
- Arrival #5 at time = 100 seconds with service time = 20 seconds

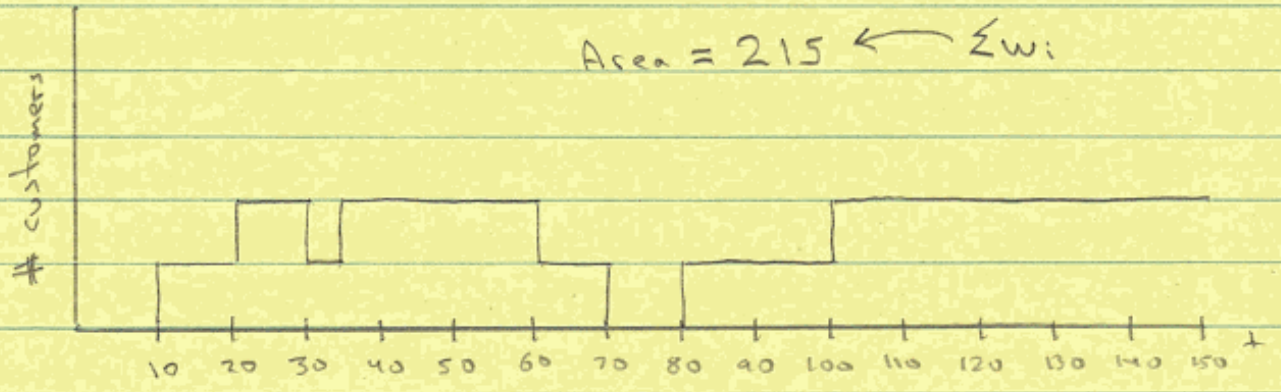
For solution see next page (scan of handwritten solution)

KJC

6/17/11

# Solution to Problem #2




$\lambda = C/T = 3/150$ customers/s

$T_S = B/C = 130/3$ s

$U = B/T = 130/150$

$W = \Sigma w_i/C = 215/3$ s

$W_q = W - T_S = 215/3 - 130/3 = 85/3$ s

$L = \Sigma w_i/T = 215/150$ customers

$L_q = L - U = 215/150 - 130/150 = 85/150$

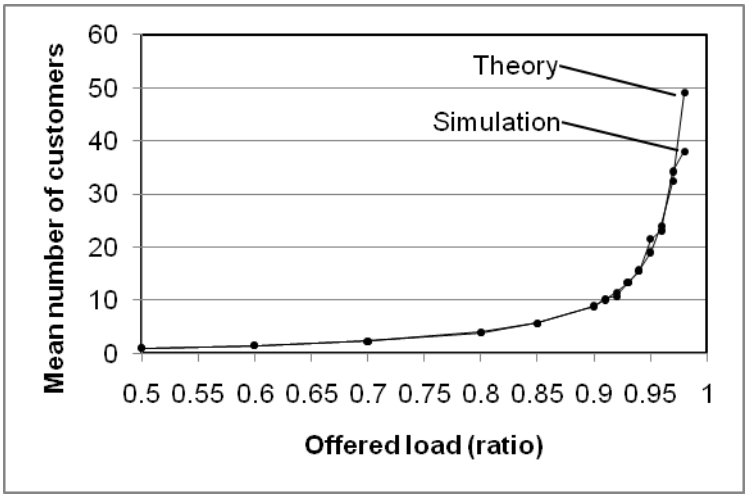
**Problem #2** (30 points)

Using the `mm1.c` simulation program we discussed in class (and that is available for download via the class website), simulate the following offered loads for an M/M/1 queue: 50%, 60%, 70%, 80%, 85%, 90%, 91%, 92%, …, 98%. Fix the service time to be 1.0. For each offered load collect results on the mean number of customers in the system (*L*). Use a SIM_TIME of 200000 seconds. Plot both the simulation results and theory results (based on the formula for *L* for M/M/1) on one graph. Plot a graph of relative error for simulation to theory versus offered load on another graph. Comment on the relative error. Does it stay the same for all offered loads?

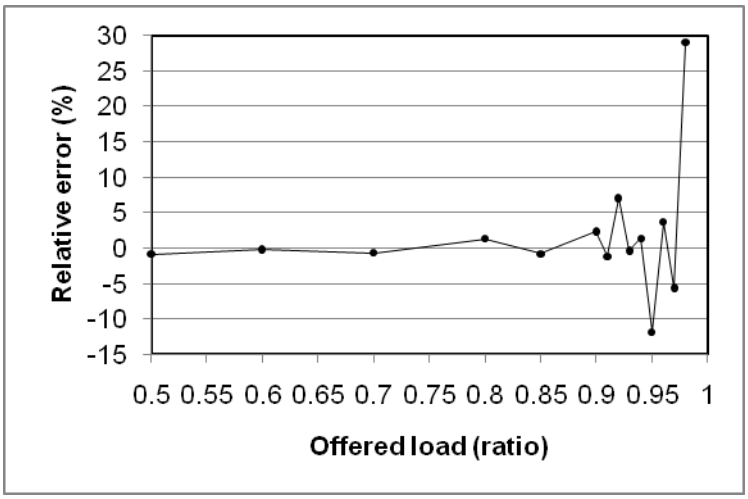
**M/M/1 simulation and theory results**

SIM_TIME = 200000 seconds

| offered | sim L | theory L | Error (%) |
|---|---|---|---|
| 0.5 | 1.008 | 1.000 | -0.82 |
| 0.6 | 1.503 | 1.500 | -0.21 |
| 0.7 | 2.349 | 2.333 | -0.67 |
| 0.8 | 3.948 | 4.000 | 1.31 |
| 0.85 | 5.713 | 5.667 | -0.82 |
| 0.9 | 8.793 | 9.000 | 2.36 |
| 0.91 | 10.231 | 10.111 | -1.17 |
| 0.92 | 10.744 | 11.500 | 7.04 |
| 0.93 | 13.337 | 13.286 | -0.38 |
| 0.94 | 15.453 | 15.667 | 1.38 |
| 0.95 | 21.541 | 19.000 | -11.80 |
| 0.96 | 23.153 | 24.000 | 3.66 |
| 0.97 | 34.251 | 32.333 | -5.60 |
| 0.98 | 37.980 | 49.000 | 29.02 |




The absolute magnitude of relative error increases as offered load increases.



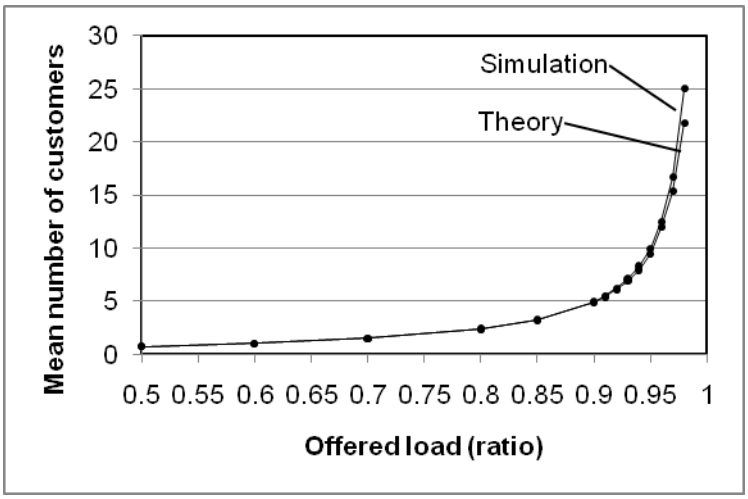
**Problem #3** (35 points)

Repeat problem #2 for M/D/1 (of course, you can't use the formula for *L* for M/M/1, you must use the P-K formula correctly). You will need to modify `mm1.c` to model an M/D/1 queue. In addition to the two plots, also submit your modified `mm1.c` (perhaps call it `md1.c`?) source code. Comments on the relative error – is it greater or smaller than for the M/M/1 simulation? Speculate on the "why".

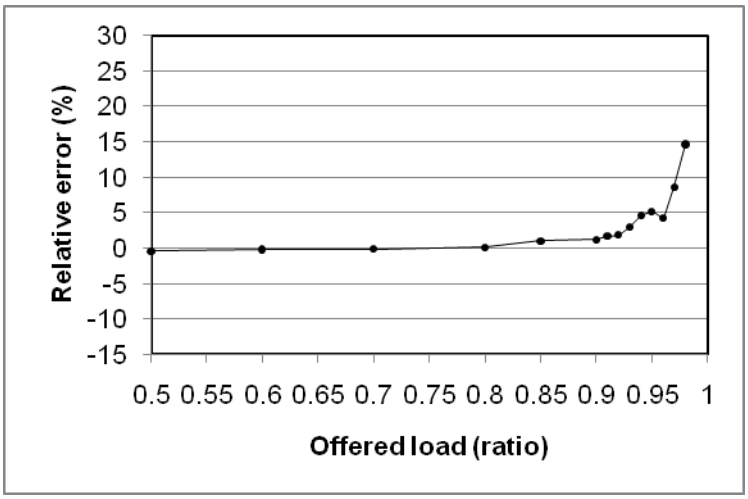
**M/D/1 simulation and theory results**

SIM_TIME = 200000 seconds

| offered | sim L | theory L | Error (%) |
|---|---|---|---|
| 0.5 | 0.753 | 0.750 | -0.45 |
| 0.6 | 1.052 | 1.050 | -0.18 |
| 0.7 | 1.518 | 1.517 | -0.12 |
| 0.8 | 2.398 | 2.400 | 0.10 |
| 0.85 | 3.224 | 3.258 | 1.05 |
| 0.9 | 4.891 | 4.950 | 1.20 |
| 0.91 | 5.421 | 5.511 | 1.65 |
| 0.92 | 6.096 | 6.210 | 1.88 |
| 0.93 | 6.906 | 7.108 | 2.93 |
| 0.94 | 7.938 | 8.303 | 4.60 |
| 0.95 | 9.482 | 9.975 | 5.20 |
| 0.96 | 11.977 | 12.480 | 4.20 |
| 0.97 | 15.327 | 16.652 | 8.65 |
| 0.98 | 21.793 | 24.990 | 14.67 |




The absolute magnitude of relative error is less than for the M/M/1 simulation. This is because there is less variability in the M/D/1 case (deterministic service, which has variance = 0) than the M/M/1 case (exponential service time with non-zero variance).



## Appendix – M/D/1 simulation program for problem #3

```
//============================================================= file = md1.c =====
//=  A simple "straight C" M/D/1 queue simulation                               =
//============================================================================
//=  Notes:                                                                    =
//=   1) The values of SIM_TIME, ARR_TIME, and SERV_TIME need to be set.       =
//=---------------------------------------------------------------------------=
//=  Build: gcc md1.c -lm, bcc32 md1.c, cl md1.c                               =
//=---------------------------------------------------------------------------=
//=  Execute: mm1                                                              =
//=---------------------------------------------------------------------------=
//=  History: KJC (06/06/11) - Genesis (from mm1.c)                            =
//============================================================================
//----- Include files -------------------------------------------------------
#include <stdio.h>              // Needed for printf()
#include <stdlib.h>             // Needed for exit() and rand()
#include <math.h>               // Needed for log()

//----- Constants -----------------------------------------------------------
#define SIM_TIME   200000.0     // Simulation time
#define ARR_TIME   (1.0/0.90)   // Mean time between arrivals
#define SERV_TIME  1.00         // Mean service time

//----- Function prototypes -------------------------------------------------
double rand_val(int seed);      // RNG for unif(0,1)
double exponential(double x);   // Generate exponential RV with mean x

//===== Main program ========================================================
int main(void)
{
  double   end_time = SIM_TIME; // Total time to simulate
  double   Ta = ARR_TIME;       // Mean time between arrivals
  double   Ts = SERV_TIME;      // Mean service time
  double   time = 0.0;          // Simulation time
  double   t1 = 0.0;            // Time for next event #1 (arrival)
  double   t2 = SIM_TIME;       // Time for next event #2 (departure)
  unsigned int n = 0;           // Number of customers in the system
  unsigned int c = 0;           // Number of service completions
  double   b = 0.0;             // Total busy time
  double   s = 0.0;             // Area of number of customers in system
  double   tn = time;           // Variable for "last event time"
  double   tb;                  // Variable for "last start of busy time"
  double   x;                   // Throughput
  double   u;                   // Utilization
  double   l;                   // Mean number in the system
  double   w;                   // Mean residence time

  // Seed the RNG
  rand_val(1);

  // Main simulation loop
  while (time < end_time)
  {
    if (t1 < t2)                // *** Event #1 (arrival) ***
    {
```

```
      time = t1;
      s = s + n * (time - tn);  // Update area under "s" curve
      n++;
      tn = time;                // tn = "last event time" for next event
      t1 = time + exponential(Ta);
      if (n == 1)
      {
        tb = time;              // Set "last start of busy time"
        t2 = time + Ts;         // NOTE: This line is modified from mm1.c
      }
    }
    else                        // *** Event #2 (departure) ***
    {
      time = t2;
      s = s + n * (time - tn);  // Update area under "s" curve
      n--;
      tn = time;                // tn = "last event time" for next event
      c++;                      // Increment number of completions
      if (n > 0)
        t2 = time + Ts;         // NOTE: This line is modified from mm1.c
      else
      {
        t2 = SIM_TIME;
        b = b + time - tb;      // Update busy time sum if empty
      }
    }
  }

  // End of simulation so update busy time sum
  b = b + time - tb;

  // Compute outputs
  x = c / time;   // Compute throughput rate
  u = b / time;   // Compute server utilization
  l = s / time;   // Compute mean number in system
  w = l / x;      // Compute mean residence or system time

  // Output results
  printf("================================================================ \n");
  printf("=            *** Results from M/D/1 simulation ***            = \n");
  printf("================================================================ \n");
  printf("=  Total simulated time         = %3.4f sec   \n", end_time);
  printf("================================================================ \n");
  printf("=  INPUTS:                                     \n");
  printf("=    Mean time between arrivals = %f sec      \n", Ta);
  printf("=    Mean service time          = %f sec      \n", Ts);
  printf("================================================================ \n");
  printf("=  OUTPUTS:                                    \n");
  printf("=    Number of completions      = %ld cust    \n", c);
  printf("=    Throughput rate            = %f cust/sec \n", x);
  printf("=    Server utilization         = %f %%       \n", 100.0 * u);
  printf("=    Mean number in system      = %f cust     \n", l);
  printf("=    Mean residence time        = %f sec      \n", w);
  printf("================================================================ \n");
  return(0);
}
```

```
//=========================================================================
//= Multiplicative LCG for generating uniform(0.0, 1.0) random numbers    =
//=   - x_n = 7^5*x_(n-1)mod(2^31 - 1)                                    =
//=   - With x seeded to 1 the 10000th x value should be 1043618065       =
//=   - From R. Jain, "The Art of Computer Systems Performance Analysis," =
//=     John Wiley & Sons, 1991. (Page 443, Figure 26.2)                  =
//=   - Seed the RNG if seed > 0, return a unif(0,1) if seed == 0         =
//=========================================================================
double rand_val(int seed)
{
  const long  a =      16807;  // Multiplier
  const long  m = 2147483647;  // Modulus
  const long  q =     127773;  // m div a
  const long  r =       2836;  // m mod a
  static long x;               // Random int value (seed is set to 1)
  long        x_div_q;         // x divided by q
  long        x_mod_q;         // x modulo q
  long        x_new;           // New x value

  // Seed the RNG
  if (seed != 0) x = seed;

  // RNG using integer arithmetic
  x_div_q = x / q;
  x_mod_q = x % q;
  x_new = (a * x_mod_q) - (r * x_div_q);
  if (x_new > 0)
    x = x_new;
  else
    x = x_new + m;

  // Return a random value between 0.0 and 1.0
  return((double) x / m);
}

//=========================================================================
//=  Function to generate exponentially distributed RVs using inverse method   =
//=    - Input:  x (mean value of distribution)                           =
//=    - Output: Returns with exponential RV                              =
//=========================================================================
double exponential(double x)
{
  double z;                    // Uniform random number from 0 to 1

  // Pull a uniform RV (0 < z < 1)
  do
  {
    z = rand_val(0);
  }
  while ((z == 0) || (z == 1));

  return(-x * log(z));
}
```