Exam #2 for Computer Networks II (CIS 6930)

Spring 2003

>>> Solutions <<<

Welcome to Exam #2 in Computer Networks II (CIS 6930). You have 75 minutes. Read each problem carefully. There are seven required problems (each problem is worth 14 points - you get 2 points for correctly following instructions). You may have with you a calculator, pencils, eraser, blank paper, lucky rabbit's foot, and one 8.5 x 11 inch "formula sheet". On this formula sheet you may have anything you want (definitions, formulas, etc.) *handwritten by you*. You may use both sides. Computer generated text, photocopies, and scans are not allowed on this sheet. Please submit your formula sheet with your exam. Please start each numbered problem on a new sheet of paper and do not write on the back of the sheets (I do not care about saving paper!). Submit everything in problem order. No sharing of calculators. Good luck and be sure to show your work.

Problem #1

Answer the following short-answer questions:

a) What is a model?

"A model is a representation (physical, logical, or functional) that mimics another object under study" (Molloy, 1989)

b) What is a model used for (i.e., what questions can we ask with a model)?

A model is used to answer questions such as "how does it work", "given this input what will be the output", and "given this output, what should the input be"

c) What is operational analysis?

Fundamental relationships that hold independent of distributions (of interarrivals and service times).

f) In order of "ugliness", what are the ways to model a system according to Kleinrock?

1) Mathematical analysis leading to a closed-form expression
2) Mathematical analysis leading to a numerical procedure
3) Simulation study
4) Experimental study (build the system)

e) Why study queueing theory?

Queueing dominates performance in computer networks. Thus, by understanding queueing we can understand the performance of computer networks.

f) What is the Markov property?

For a system of state, the past history is completely summarized in the current state.

Problem #2

Completely describe the Kendall notation for queues.

A queue is described as A/S/c/k/m where A is the arrival distribution, S the service distribution, c the number of servers, k the capacity of the queue, and m the number of customers in the system. A and S can be "M" for Markovian, "D" for determinisitic, "G" for general, and so on. Any numerical that is inifinity is omitted (e.g., M/M//100 has one server and infinite queue capacity).

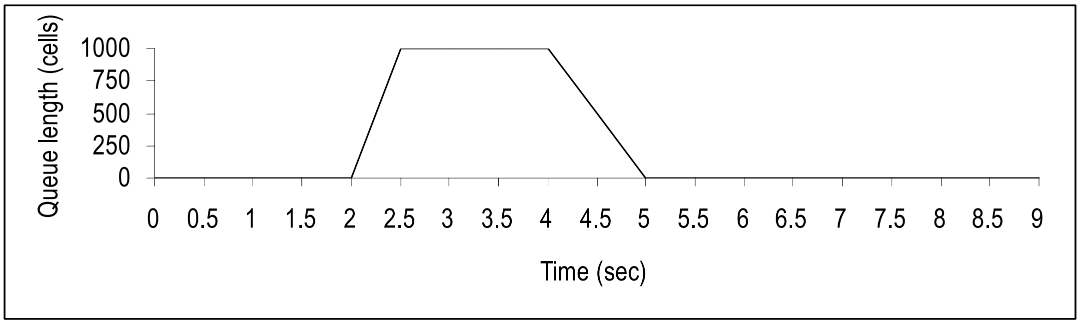
Problem #3

Consider an ATM switch with a cell switching rate of 10,000 cells per second. Cells are a fixed length. Assume that the ATM switch has a buffer of capacity 1000 cells. Using fluid-flow assumptions, plot the queue length (buffer length) as a function of time for the following input (assume that at time 0 the queue contains zero cells):

- From time 0 to 2 seconds the input rate is 5,000 cells per second
- From time 2 to 4 seconds the input rate is 12,000 cells per second
- From time 4 to 9 seconds the input rate is 9,000 cells per second

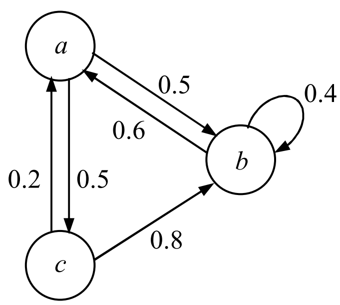
From time 0 to 9 seconds, about how many cells are lost due to buffer overflow? Show your work!

From time 0 to 2 seconds the arrival rate is less than the service rate, so the queue length remains 0 cells. From time 2 to 4 seconds the queue is increasing in length at a rate 2,000 cells per second. After 0.5 second (at time = 2.5 seconds) the queue is full. Thus, from time 2.5 to 4 seconds 3,000 cells are lost. From time 4 to 9 seconds the queue is decreasing at 1,000 cells per second and thus empties out at time 5 (and from time 5 to 9 the queue is empty). The plot is:



Problem #4

Solve for the steady-state probabilities of the three-state Markov chain below(with states *a*, *b*, and *c* and transition probabilities as shown). Show your work.



The probability matrix is:

$$P := \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.6 & 0.4 & 0 \\ 0.2 & 0.8 & 0 \end{pmatrix}$$

The equations to be solved are:

$$p0 = 0 \cdot p0 + 0.6 \cdot p1 + 0.2 \cdot p2$$

$$p1 = 0.5 \cdot p0 + 0.4 \cdot p1 + 0.8 \cdot p2$$

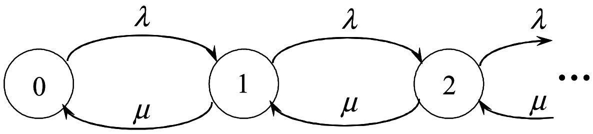
$$1 = p0 + p1 + p2$$

From these equations the steady state probabilities are p0 = 1/3. p1 = 1/2, and p2 = 1/6.

Problem #5

For an M/M/1 queue derive the steady state probability for k customers in the system (πk), then derive the mean number in the system (L), mean system delay (W), mean number if queue (Lq), and mean wait in qeueu (Wq) for an M/M/1 queue. Show your work.

The transition state diagram for the M/M/1 is:



From flow balance we write:

$$\lambda\pi_0 = \mu\pi_1$$
$$\lambda\pi_1 = \mu\pi_2$$
$$\vdots$$
$$\lambda\pi_n = \mu\pi_{n-1}$$

Which is the same as:

$$\pi_1 = \left(\frac{\lambda}{\mu}\right)\pi_0$$
$$\pi_2 = \left(\frac{\lambda}{\mu}\right)\pi_1 = \left(\frac{\lambda}{\mu}\right)^2 \pi_0$$
$$\vdots$$
$$\pi_n = \left(\frac{\lambda}{\mu}\right)^n \pi_0$$

We have that $\rho = \lambda/\mu$ and thus $\pi_n = \rho^n \pi_0$. We can solve for π_0 from the fact that $\sum_{i=0}^{\infty}\pi_n = 1$. That is,

$$\pi_0 = \frac{1}{\sum_{n=0}^{\infty}\rho_n} = \frac{1}{1+\sum_{n=1}^{\infty}\rho_n} = \frac{1}{1+\dfrac{\rho}{1-\rho}} = 1-\rho$$

And thus, $\pi_n = (1-\rho)\rho^n$. To solve for L we find the mean:

$$L = \sum_{n=0}^{\infty} n\pi_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n = \frac{\rho}{1-\rho}$$

And we have the rest quite easily:

$$W = \frac{L}{\lambda} = \frac{1}{\mu-\lambda}$$

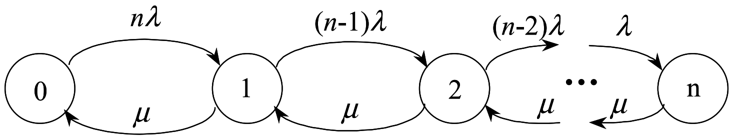
$$Lq = L-\rho = \frac{\rho^2}{1-\rho}$$

$$Wq = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$$

Problem #6

Derive an expression (closed form not needed) for steady-state probablity (πk) for the M/M/1//n queue.

This problem comes directly from assignment #5. The transition state diagram is:



We write:

$$
\begin{aligned}
n\lambda\pi_0 &= \mu\pi_1 \\
(n-1)\lambda\pi_1 &= \mu\pi_2 \\
&\vdots \\
\lambda\pi_{n-1} &= \mu\pi_n
\end{aligned}
$$

From which we can see…

$$
\begin{aligned}
\pi_1 &= n\rho\pi_0 \\
\pi_2 &= (n-1)\rho\pi_1 = n(n-1)\rho^2\pi_0 \\
&\vdots \\
\pi_n &= \rho\pi_{n-1} = n!\,\rho^n\pi_0
\end{aligned}
$$

Thus,

$$\pi_k == \frac{n!}{(n-k)!}\rho^k\pi_0$$

Problem #7

Consider a system with 3 servers (each with service rate $\mu = 1.0$) and Poisson arrivals of rate $\lambda = 0.80$. If the system has no queue (i.e., it only has 3 servers), compute the probability of an arriving customer being blocked (and thus lost) – that is, compute Pr[blocked]. If the system has an infinite length queue, compute the probability of an arriving customer being queued – that is, compute Pr[queued]. Which is greater, Pr[block] or Pr[queue]? Explain why.

This problem is asking for Erlang-B and Erlang-C probabilities:

$$\Pr[block] = \frac{\left(\frac{\lambda}{\mu}\right)^m\left(\frac{1}{m!}\right)}{\sum_{n=0}^{m}\left(\frac{\lambda}{\mu}\right)^n\left(\frac{1}{n!}\right)}$$ is Erlang-B and $$\Pr[queue] = \frac{\left(\frac{\lambda}{\mu}\right)^m\left(\frac{1}{m!}\right)}{\left(1-\frac{\lambda}{\mu}\right)\sum_{n=0}^{m-1}\left(\frac{\lambda}{\mu}\right)^n\left(\frac{1}{n!}\right)+\left(\frac{1}{m!}\right)\left(\frac{\lambda}{\mu}\right)^m}$$ is Erlang-C

For $m = 3$ and $\frac{\lambda}{\mu} = 0.80$ we get Pr[block] = 0.039 and Pr[queue] = 0.168. Pr[queue] > Pr[block] because arriving customers may arrive to a queued customer whereas in the Erlang-B case there will never be a customer not-in-service to wait behind (and thus the probability or arriving to an empty server is higher).
