Exam #2 for Performance Evaluation (CIS 6930)

Fall 2004

>>> Solutions <<<

Welcome to Exam #2 in Performance Evaluation (CIS 6930). You have 75 minutes. Read each problem carefully. There are eight required problems (each problem is worth 12 points - you get 4 points for correctly following instructions). You may have with you a calculator, pencils, eraser, blank paper, lucky rabbit's foot, and one 8.5 x 11 inch "formula sheet". On this formula sheet you may have anything you want (definitions, formulas, etc.) *handwritten by you*. You may use both sides. Computer generated text, photocopies, and scans are not allowed on this sheet. Please submit your formula sheet with your exam. Please start each numbered problem on a new sheet of paper and do not write on the back of the sheets (I do not care about saving paper!). Submit everything in problem order. No sharing of calculators. Good luck and be sure to show your work.

Problem #1

Answer the following short-answer questions:

a) What is a model?

> "A model is a representation (physical, logical, or functional) that mimics another object under study" (Molloy, 1989)

b) What is the goal of a model and why do we build models?

> The goal of a model is to be able to predict some behavior of the orginal system. Building models is one or more of cheaper, easier, faster, safer than building and experimenting on an actual system.

c) Define the Markov property

> The probability of transition to the next state is only a function of the current state and not of the history of previous states visited. The current state "holds" the history of all previous states.

d) In one sentence describe the key finding of the Jain Packet Trains paper.

> The key contribution was to show that network packet traffic is not Poisson (i.e., interarrival times are not exponentially distributed), but instead, packets arrive in correlated bursts that Jain characterized as "packet trains".

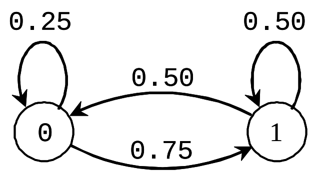
e) In one sentence describe the key finding of the Leland Bellcore paper.

> The key contribution was to show that network packet traffic was self-similar (long range dependent, fractal) and to show that such traffic does not "smooth out" when aggregated.

Problem #2

You are given a probability matrix, *P*, below for a Markov chain. Draw the Markov chain for this probability matrix. Solve for the steady state probabilities for the two states (numbered 0 and 1).

$$P = \begin{bmatrix} 0.25 & 0.75 \\ 0.50 & 0.50 \end{bmatrix}$$



We can write two equations in two unknowns:

$$\pi_0 = 0.25\pi_0 + 0.50\pi_1$$
$$1 = \pi_0 + \pi_1$$

Solving for this we get $\pi_0 = 0.40$ and $\pi_1 = 0.60$.

Problem #3

For the *P* matrix given above assume that the system starts in state 0, in three time steps what is the probability that the system is in state 1?

We iterate three times $\pi(n+1) = \pi(n) \cdot P$ and starting with $n = 1$ and $\pi(1) = [1 \quad 0]$ and get $\pi(3) = [0.3906 \quad 0.6094]$. Thus, the probability of being in state 1 in three time steps is 0.6094.

Problem #4

For an M/M/2/2 queue with mean service time 1.0 seconds for each server and arrival rate 1.5 customers per second, what is the probability that an arrival will be blocked (i.e., not able to enter service)? Hint: Service rate (μ) is the reciprocal of mean service time.

This is an Erlang-B problem. The Erlang-B formula is

$$\Pr[block] = \frac{\left(\frac{\lambda}{\mu}\right)^m \left(\frac{1}{m!}\right)}{\sum_{n=0}^{m} \left(\frac{\lambda}{\mu}\right)^n \left(\frac{1}{n!}\right)}$$

Solving this for $\lambda = 1.5$, $\mu = 1$, and $m = 2$ we get $\Pr[block] = 0.3103$.

Problem #5

What is the mean response time (mean delay in system) for an M/unif(*a*, *b*)/1 queue where unif(*a*, *b*) is a uniform distribution from *a* to *b* seconds. The arrival rate is λ customers per second. The parameter values are $\lambda = 1$, $a = 0$, and $b = 1$. Hint: The mean for a unif(*a*, *b*) is $(a+b)/2$ and the variance is $(a-b)^2/12$. That is, the mean service time of a M/unif(*a*, *b*)/1 queue is $(a+b)/2$ seconds.

This is an M/G/1 problem where the P-K formula is $W = \frac{\lambda \cdot E[x^2]}{2 \cdot (1-\rho)} + E[x]$ where $E[x]$ is the mean service time and $E[x^2]$ is the second moment of the service time (where $E[x^2] = \sigma^2 + E[x]^2$ for σ^2 variance). We have $\lambda = 1$, $E[x] = \frac{1+0}{2} = 0.5$ (note that μ is the reciprocal of $E[x]$ and is thus $\mu = 2$ in this case), $E[x^2] = \frac{(0-1)^2}{12} + 0.5^2 = 1/3$, and $\rho = \frac{1}{2} = 0.5$. Thus, $W = \frac{1 \cdot (1/3)}{2 \cdot (1-0.5)} + 0.5 = 5/6$ seconds.

Problem #6

Answer the following short-answer questions:

a) Precisely define what is a simulation.

> "A software program that models the imporant aspects of a system under study. Or, "the dicipline of designing a model of an actual or theoretical physical system, executing the model on a computer, and analyzing the execution output.: (Fishwick, 1989)."

b) Describe the phases of a simulation study

> Design, Execution, and Output Analysis are the three phases. Each phase is inter-related with every other phase.

c) List the components of a simulation model

> System state, simulation clock, event list, event routine, statistical counters, library routines, and report generator.

d) List the four desired properties of a random number generator

> The four desired properties are:
> 1. Numbers produced must be uniformly distributed and exhibit no correlation
> 2. Generator must be fast and not require much storage
> 3. Random number stream must be reproducible
> 4. Must be able toproduce separate streams of random numbers

Problem #7

What is the output for the below CSIM program?

```
#include <stdio.h>
#include "csim.h"

void process1(void);

void sim(void)
{
  int i;

  create("sim");
```

```
  printf("start at %f \n", clock);
  process1();
  hold(0.25);
  for (i=0; i<4; i++)
  {
    hold(0.75);
    printf("sim at %f \n", clock);
  }
}

void process1(void)
{
  create("process1");

  while(1)
  {
    hold(1.0);
    printf("process1 at %f \n", clock);
  }
}
```

The output is:

```
start at 0.000000
process1 at 1.000000
sim at 1.000000
sim at 1.750000
process1 at 2.000000
sim at 2.500000
process1 at 3.000000
sim at 3.250000
```

Problem #8

Is system A really better (better = lower response time) than system B? Argue why or why not. The table below gives five response time measurements for systems A and B. You may assume that the measurements are from a representative sample. A T-score table is given below.

Experiment #	Response time for System A	Response time for System B
1	110 ms	110 ms
2	90	105
3	100	110
4	100	108
5	101	102

We first find D to be 0, 15, 10, 8, and 1. The mean of D is 6.8 and the sample standard deviation of D is 6.301. From the T-score table for 4 degrees of freedom we find the $t_{\alpha/2}$ for 95% CI is 2.78 and for 90% CI it is 2.13. Thus, H95 (half-width for 95% CI) is 2.78 * (6.301 / sqrt(5)) = 7.833 and H90 = 2.13 * (6.301 / sqrt(5)) = 6.002. Thus, for 95% CI we have the true mean in a range of (6.8 - 7.833) to (6.8 + 7.833) which crosses zero, so we cannot say with 95% confidence that one system is better than another. For 90% CI we have the true mean in a range of (6.8 - 6.002) to

(6.8 + 7.833) which is entirely above zero, so we can say with 90% confidence that system A is better (lower mean delay). The question then is are we happy with 90% confidence, or must we have 95% confidence (or perhaps an even high confidence)? If we are happy with 90% confidence, then System A can be deemed better, otherwise it cannot be deemed better.

T-score table

***N* – 1**	**t for $\alpha/2$ = 0.050**	**t for $\alpha/2$ = 0.025**
4	2.13	2.78
5	2.02	2.57
6	1.94	2.45
7	1.90	2.37
8	1.86	2.31
9	1.83	2.26
10	1.81	2.23
11	1.80	2.20
12	1.78	2.18
13	1.77	2.16
